Math 2144 Final **Fall 2022** **NAME:** ____________________

CRN: ____________________

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an approved calculator to perform operations on real numbers, evaluate functions at specific values, and look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e., laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work**, including all steps needed to solve each problem and explain your reasoning to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 9 problems on 13 pages (including the page containing basic formulas). Make sure all problems and pages are present.

The exam is worth 86 points in total.

You have **1 hour** and **50 minutes** to work starting from the signal to begin.

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1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Suppose the **derivative** of f is given by $\dfrac{df}{dx} = e^{\cos(\pi x)}$. Using the linearization of f at $x = 5$, which of the following is the best approximation of $f(5.3)$?

   a. $f(5.3) \approx f(5) + 0.3e^{\cos(5\pi)}$

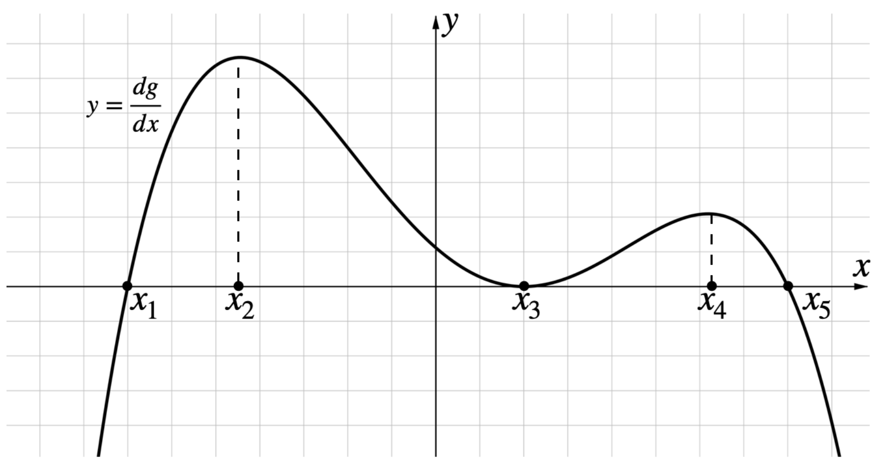
   b. $f(5.3) \approx e^{\cos(\pi x)} \cdot f(5)$

   c. $f(5.3) \approx \dfrac{0.3e^{\cos(5\pi)}}{f(5)}$

   d. $f(5.3) \approx f(5) + e^{\cos(5\pi)}$

   e. $f(5.3) \approx 0.3f(5) + e^{\cos(5\pi)}$

   (ii) The graph of the **derivative** of g, $y = g'(x)$, is shown below. Identify which of the following statements are true.



   I. x_2 and x_4 are critical points of g.

   II. $g(x_3)$ is a local extreme value (either maximum or minimum) of g.

   III. x_1, x_3, and x_5 are critical points of g.

   IV. $(x_2,\ g(x_2))$, $(x_3,\ g(x_3))$, and $(x_4,\ g(x_4))$ are inflection points of g.

   V. $g(x_1)$ is a local minimum of g.

   a. I only

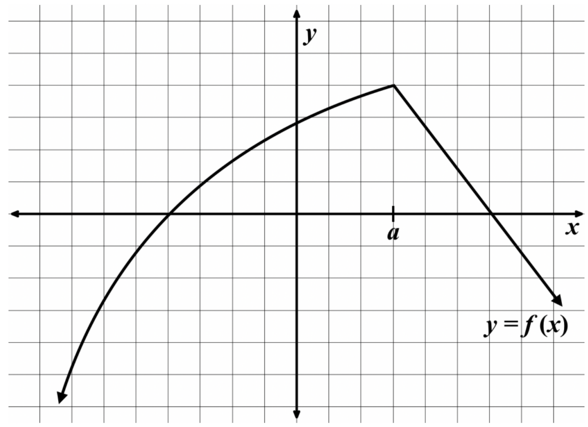
   b. I and II only

   c. III and V only

   d. II and III only

   e. III, IV, and V only

(iii) The graph of the function $y = f(x)$ is shown below.



Which statement below is true? (Select only one)

a. The function $y = f'(x) = \frac{df}{dx}$ is continuous at $x = a$.

b. $\lim_{x \to a} \frac{df}{dx}$ exists.

c. The function $y = f'(x) = \frac{df}{dx}$ has a jump discontinuity at $x = a$.

d. $f'(a) = \left.\frac{df}{dx}\right|_{x=a} = 0$.

e. The function $y = f'(x) = \frac{df}{dx}$ has a removable discontinuity at $x = a$.

(iv) Yvonne decides to go for a run before school. She starts her run from home. The function $y = v(t)$ expresses Yvonne's velocity (in meters per minute) t minutes after she started running. What quantity does the following sum approximate?

$$\sum_{k=1}^{6} v\left(4 + \frac{k-1}{2}\right) \cdot \frac{1}{2}$$

a. The average rate of change of Yvonne's velocity over the interval of time from $t = 1$ to $t = 4$.

b. The change in Yvonne's distance away from home over the interval of time from $t = 4$ to $t = 7$.

c. Yvonne's acceleration over the interval of time from $t = 1$ to $t = 6$.

d. Yvonne's distance away from home after having run for 2.5 minutes.

e. Yvonne's instantaneous velocity 6.5 minutes after having left home.

(v) Suppose $\int_{-2}^{9} f(x)\,dx = 12$ and $\int_{3}^{9} f(x)\,dx = 4$. What is $\int_{-2}^{3} f(x)\,dx$?

a. 16

b. -8

c. 3

d. 8

e. -16

2. (*4 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numerical answers without justification will earn no credit. **If you use L'Hôpital's Rule, clearly justify why you are able to do so**.

(a) $\lim_{x\to 0} \frac{1-\cos(x)}{x} =$

(b) $\lim_{\theta\to \frac{\pi}{2}} \frac{\tan(\theta)}{\sec(\theta)} =$

3. (*4 points each*) Compute the following derivatives.

   (a) Let $g(x) = e^{x\sin(x)}$. Find $g'(x)$.

   (b) Let $f(x) = -\ln(\cos(x))$. Find the **second derivative** of f, $\dfrac{d^2 f}{dx^2}$.

   (c) Find $\dfrac{dy}{dx}$ for the curve $e^y - x^2 y = 5$. Your answer for $\dfrac{dy}{dx}$ should contain both x and y.

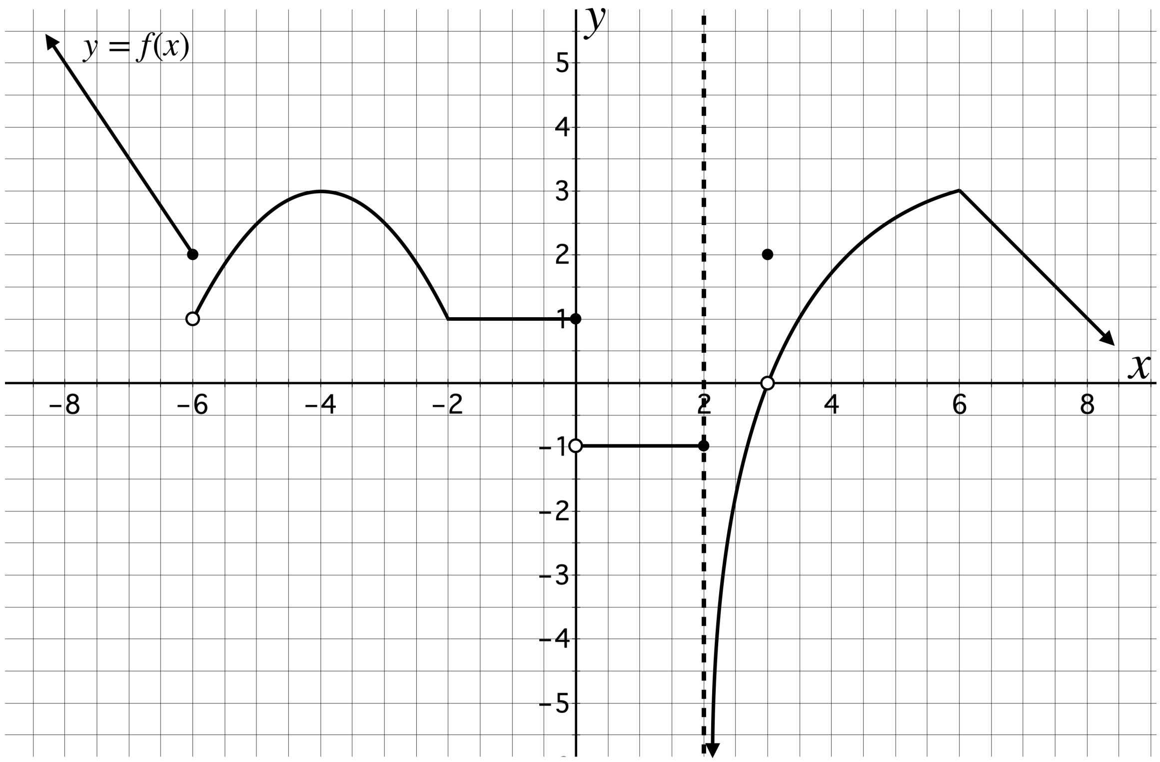
4. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

   (a) $\displaystyle\int \left(\sec^2(t) - 9t^2\right) dt =$

   (b) $\displaystyle\int_2^3 x \cdot e^{x^2}\, dx =$

   (c) $\displaystyle\frac{d}{dx}\int_1^{x^4} \cos(2\theta + 1)\, d\theta =$

5(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

$\lim_{x\to -2} f(x) =$

$\left.\frac{df}{dx}\right|_{x=-7} =$

$\int_{-2}^{2} f(x)\,dx =$

$f'(-4) =$

$\lim_{x\to -6^+} f(x) =$

$\lim_{\Delta x\to 0} \frac{f(8+\Delta x)-f(8)}{\Delta x} =$

$\lim_{x\to 2^+} \frac{df}{dx} =$

$\lim_{x\to 3} f(x) =$

$\lim_{N\to\infty} \sum_{k=1}^{N} f\left(1+\frac{k}{N}\right)\cdot\frac{1}{N} =$

5(b) (*2 points*) Identify all values of x in the interval $(-8, 8)$ where f is continuous but not differentiable.

6. (*5 points*) The function $f(t) = \ln(t+4) - \ln(4)$ gives the amount of rainfall (in inches) that accumulated between midnight and t hours after midnight. At what rate is rain falling (in inches per hour) at 2:30 AM?

7. (*4 points each*) Courtney's velocity (in miles per hour) while running a marathon is given by the function $y = v(t)$, where t represents the number of hours elapsed since Courtney started the marathon.

   (a) Write a sentence explaining the meaning of the expression $\int_1^3 v(t)\, dt$.

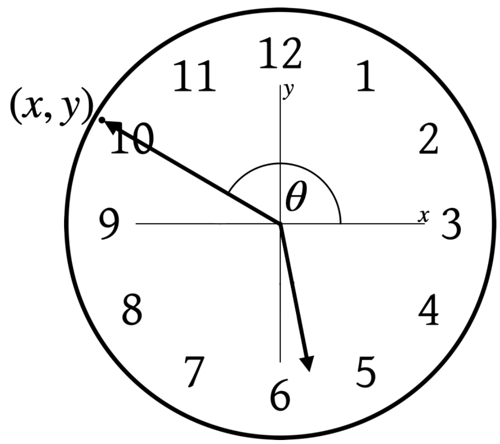
   (b) A marathon is (approximately) 26.2 miles. Given that the integral below equals 26.2, write a sentence explaining the meaning of x in the context of the situation.

$$\int_0^x v(t)\, dt = 26.2$$

8. (*10 points*) A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. At 2:00 PM, another boat is 10 km directly west of the same dock and approaches the dock heading due east at a speed of 10 km/h. At what time are the two boats **closest** together? Justify that the boats are closest together at the time you identified.

9. (*10 points*) The tip of a clock's minute hand is 9 inches from the center of the clock. Imagine a standard coordinate system with the origin at the center of the clock, as shown below. Let x and y respectively represent the horizontal and vertical coordinates of the tip of the minute hand. Find the values of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ when this exam ends (i.e., at 5:50 PM). Express your solution in units of *inches per hour*.

   (Hint: Express x and y in terms of θ.)



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BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C$$

$$\int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C$$

$$\int \sec(u)\tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\cot(u)\,du = -\csc(u) + C$$

$$\int \tan(u)\,du = \ln|\sec u| + C$$

$$\int \cot(u)\,du = \ln|\sin u| + C$$

$$\int \sec(u)\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc(u)\,du = \ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int f\left(u(x)\right)\cdot u'(x)\,dx = \int f(u)\,du$$

$$L_n = \Delta x\sum_{k=1}^{n} f\left(a + (k-1)\Delta x\right)$$

$$R_n = \Delta x\sum_{k=1}^{n} f\left(a + k\Delta x\right)$$

$$M_n = \Delta x\sum_{k=1}^{n} f\left(a + \frac{2k-1}{2}\Delta x\right)$$