Math 2144 Exam 1 **Fall 2024** **NAME:**

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an approved calculator to

- perform operations on real numbers,
- evaluate functions at specific values, and
- generate and examine at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and, when prompted, explain your reasoning to earn full credit. For those tasks that explicitly prompt you to show work or explain your reasoning, answers alone will receive no credit. The purpose of this assessment is for you to demonstrate what you know.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 6 problems on 9 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 58 points in total.

You have **60 minutes** to work starting from the signal to begin.

This page is intentionally blank. You may use it for scratch work, but any work recorded on this page will not be graded.

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) The functions f, g, and h are defined as follows:

   $$f(x) = \frac{x^2-1}{x-1} \qquad g(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases} \qquad h(x) = x+1$$

   Which of the following is true?

   a. $\lim\limits_{x\to 1} g(x) = g(1)$

   b. $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} g(x) = \lim\limits_{x\to 1} h(x)$

   c. $f(1) = g(1) = h(1)$

   d. The functions f and h are continuous at $x = 1$, but the function g has a removable discontinuity at $x = 1$.

   e. $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} h(x) \neq \lim\limits_{x\to 1} g(x)$

   (ii) An object is moving forward along a straight line. The distance the object has traveled (in meters) from its starting position t seconds after it started moving is given by $s(t) = 2\sqrt{t}$. What is the average velocity of the object over the interval from $t = 4$ to $t = 9$?

   a. 1   b. 2   c. $\frac{5}{2}$ d. $\frac{2}{5}$ e. 5

   (iii) If f and g are continuous functions with $f(5) = 5$ and $\lim\limits_{x\to 5} (2f(x) - g(x)) = 6$, find $g(5)$.

   a. $g(5) = 4$

   b. $g(5) = 5$

   c. $g(5) = 16$

   d. $g(5) = -1$

   e. $g(5) = 6$

(iv) The temperature T(in degrees Fahrenheit) of a 12 ounce cup of coffee at time t (in minutes since the coffee was brewed) is given by $T(t) = \frac{3}{8}t^2 - 13t + 180$ for $0 \leq t \leq 10$. What is the most appropriate interpretation of the statement

$$T'(4.2) = \left.\frac{dT}{dt}\right|_{t=4.2} = -9.85?$$

a. 4.2 minutes after the coffee was brewed, the temperature of the coffee was 9.85 degrees Fahrenheit less than the initial brew temperature.

b. The temperature of the coffee decreased by 9.85 degrees Fahrenheit during the fourth minute after the coffee was brewed.

c. On average, the temperature of the coffee decreased by 9.85 degrees Fahrenheit each minute over the first 4.2 minutes since it was brewed.

d. 4.2 minutes after the coffee was brewed, the temperature of the coffee was changing at a rate of -9.85 degrees Fahrenheit per minute.

e. The temperature of the coffee was approximately 132.01 degrees Fahrenheit exactly 4.2 minutes after the coffee was brewed.

(v) Alessandra is traveling down Interstate 35 at a **constant velocity**. For every increase of ten minutes in the number of minutes x since she passed mile marker 174, her distance y (in miles) from Downtown Oklahoma City decreases by seven miles. Which of the following statements is definitely true?

a. $\Delta y = -0.7\Delta x$

b. $y = -0.7x$

c. $x \approx -1.43y$

d. Statements (a) and (b) are definitely true

e. Statements (a), (b), and (c) are definitely true

2. (*3 points each*) Pistol Pete was cheering on the Cowgirl basketball team and threw a replica team jersey into the crowd. The height of the jersey above the arena floor, measured in feet, was given by the function $h(t)$ where t was the time in seconds after he threw the jersey. Describe the meaning of the following expressions in the context of this situation. Your response should identify the *quantity* represented by the expression. **Specify the units** associated with each expression.

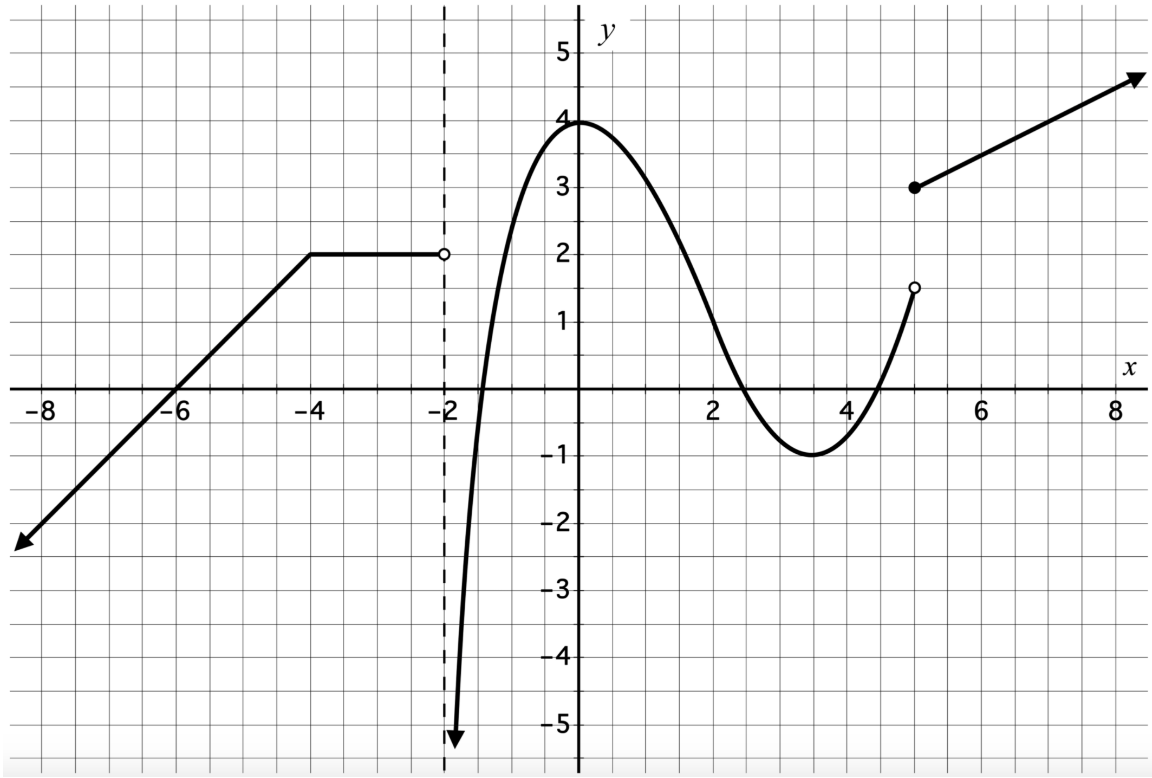
   (a) $h(2) - h(1)$

   (b) $\dfrac{h(2.5) - h(1)}{1.5}$

   (c) $\displaystyle\lim_{\Delta t \to 0} \frac{h(1 + \Delta t) - h(1)}{\Delta t}$

   (d) What does the solution to the equation $h'(t) = -3.2$ represent?

3(a). (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all maxima and minima, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. If the limit or derivative does not exist, write "∞", "−∞", or "DNE", whichever is most appropriate.

$\lim\limits_{x\to 5} f(x) =$ $\qquad$ $\left.\dfrac{df}{dx}\right|_{x=-5} =$ $\qquad$ $f^{(}5) =$

$\lim\limits_{x\to -2^-} f(x) =$ $\qquad$ $\lim\limits_{x\to -\pi} \dfrac{f(x) - f(-\pi)}{x + \pi} =$ $\qquad$ $\lim\limits_{x\to -4^-} \dfrac{df}{dx} =$

$\lim\limits_{\Delta x\to 0} \frac{f(-4+\Delta x)-f(-4)}{\Delta x} =$ $\qquad$ $f'(7) =$ $\qquad$ $\dfrac{f(7.3) - f(5)}{7.3 - 5} =$

3(b). (*1 point*) List all values of x in the interval $[-8, 8]$ for which f is continuous but not differentiable.

4. (*4 points each*) Evaluate the following limits as exact values, or state that the limit does not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. Evaluating the limits by using a table of values or examining a graph will receive no credit.

   (a) $\displaystyle\lim_{x\to 4} \frac{x^3 - 16x}{x - 4} =$

   (b) $\displaystyle\lim_{x\to 2} \frac{x^2 + 4x - 12}{x - 2} =$

5. (*6 points*) Use the limit definition of derivative to show that

$$\frac{d}{dx}\left(4 - \frac{1}{x}\right) = \frac{1}{x^2}.$$

You **must** use the **limit definition of derivative** to receive credit.

6. (*4 points each*) Compute the following derivatives. (You **may** use the Power Rule to compute these derivatives.) You do **not** need to simplify your answers.

   (a) Let $f(x) = -3x^9 - x^4$. Find $f'(x)$.

   (b) Let $y = \dfrac{5}{x^2} + 8\sqrt{x}$. Find $\dfrac{dy}{dx}$.

   (c) Let $g(t) = \dfrac{1}{t^4} - t^e + \pi$. Find $\dfrac{dg}{dt}$.