Math 2144 Exam 2 **Fall 2023** **NAME:** ______________________

CRN: ______________________

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an approved calculator to

- perform operations on real numbers,
- evaluate functions at specific values, and
- generate and examine at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and, when prompted, explain your reasoning to earn full credit. For those tasks that explicitly prompt you to show work or explain your reasoning, answers alone will receive no credit. The purpose of this assessment is for you to demonstrate what you know.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 9 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 60 points in total.

You have **60 minutes** to work starting from the signal to begin.

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1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Let $L(x) = f(6) + f'(6)\,(x-6)$ represent the linearization of f at $x = 6$. Which of the following guarantees that $L(6.5) > f(6.5)$?

   a. $f'(x) > 0$ for $6 < x < 6.5$

   b. $f'(x) < 0$ for $6 < x < 6.5$

   c. $f''(x) > 0$ for $6 < x < 6.5$

   d. $f''(x) < 0$ for $6 < x < 6.5$

   e. $f'(6.5) = 0$

   f. $f''(6.5) = 0$

   (ii) Suppose f is a twice differentiable function with critical points $x = 2$, $x = 7$, and $x = 15$. Consider the following information about the **second derivative** of f.

x	$(-\infty,\ 4)$	$(4,\ 7)$	$(7,\ 13)$	$(13,\ \infty)$
$f''(x)$	$-$	$+$	$-$	$+$

   What is the absolute (global) maximum of f on the interval $[4,\ 13]$?

   a. $f(2)$

   b. $f(4)$

   c. $f(7)$

   d. $f(13)$

   e. $f(15)$

   f. Information about the sign of the first derivative of f is necessary to answer this question.

(iii) Given that $f'(-4) = 0$ and $f''(x) = 2\sin(\pi x)$. Which statement below is true? (Only one of these statements is true.)

a. f has a local minimum at $x = -4$

b. f has a local maximum at $x = -4$

c. f has a point of inflection at $x = -4$

d. f is increasing at $x = -4$

e. f is decreasing at $x = -4$

(iv) A lighthouse is located on a small island 3 kilometers away from the nearest point P on a straight shoreline. Let x represent the distance between P and the light beam's intersection with the shoreline. Also let θ represent the measure of the angle created by the beam of light and the line connecting the lighthouse and P. Which formula defines the relationship between the rate of change of the angle's measure and the rate at which the beam of light is moving along the shoreline?

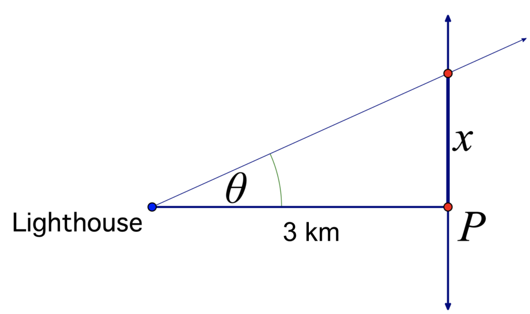
a. $\tan\left(\dfrac{d\theta}{dt}\right) = \dfrac{dx}{dt}$

b. $\sec^2(\theta) \cdot \dfrac{d\theta}{dt} = \dfrac{1}{3} \cdot \dfrac{dx}{dt}$

c. $\sec^2\left(\dfrac{d\theta}{dt}\right) = \dfrac{1}{3} \cdot \dfrac{dx}{dt}$

d. $\dfrac{d\theta}{dt} = \dfrac{1}{3\sec^2(\theta)}$

e. $\dfrac{d\theta}{dt} = \dfrac{1}{1 + \left(\frac{dx}{dt}\right)^2}$



(v) Suppose the function $g(t) = 6t^2 - 0.75t^3$ represents the height of a golfball above the ground (in yards) t seconds since the ball was struck (only for values of t between 0 and 8). By solving which of the following equations can we determine the value of t for which the height of the ball above the ground is maximized?

a. $g'(t) = 12t - 2.25t^2$

b. $0 = 6t^2 - 0.75t^3$

c. $g(t) = 6t^2 - 0.75t^3$

d. $0 = 12t - 2.25t^2$

e. $g''(t) = 12 - 4.5t$

2. (*8 points*) Use calculus to determine the following for the function

$$f(x) = 3x^4 - 24x^3 + 54x^2.$$

(a) (*4 points*) Find all critical points of f. Work must be shown to receive credit.

(b) (*4 points*) Determine the absolute (global) maximum and absolute (global) minimum value of $f(x)$ on the interval $[-1, 4]$. Work must be shown to receive credit.

3. (*4 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so**.

   (a) $\displaystyle\lim_{\theta\to\pi}\left(\frac{\sin(\theta)}{e^{\tan(\theta)}+\cos(\theta)}\right) =$

   (b) $\displaystyle\lim_{x\to 5}\left(\frac{\ln\left(\frac{x}{5}\right)}{\pi^{x-5}}\right) =$

4. (*4 points each*) Compute the following derivatives.

   (a) Let $f(x) = \dfrac{7^x}{x^7}$. Find $f'(x)$.

   (b) Let $y = \ln(\sec(x))$. Find the **second derivative** $\left(\frac{d^2y}{dx^2}\right)$, or y'', of y with respect to x.

5. (*6 points*) Find $\dfrac{dy}{dx}$ for the curve $\ln\left(3x^4\right) = e^{\sin(y)} + y$. Solve for $\dfrac{dy}{dx}$ in terms of both x and y.

6. (*10 points*) Firefighter Everett raises the ladder of a firetruck to the third floor of a burning building. As the ladder elevates, the measure of the angle, θ, the ladder makes with the horizontal changes at a constant rate of $\frac{\pi}{20}$ radians per second. At what rate does the slope of the ladder change when $\theta = \frac{\pi}{6}$?

   Express your solution as an exact value, not an approximation.

   (*Hint: Consider which of* $\sin(\theta)$*,* $\cos(\theta)$*, or* $\tan(\theta)$ *represents the slope of the ladder.*)

7. (*10 points*) A boat leaves a dock at noon and travels due north at a speed of 5 kilometers per hour. Another boat has been heading due east at 10 kilometers per hour and reaches the same dock at 2:00 PM. At what time were the two boats closest to each other?

BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$