Math 2144 Exam 2 **Spring 2023** **NAME:** ____________________

CRN: ____________________

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5 inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. Reasoning which will earn credit will use material covered in the course to date.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 6 problems on 9 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 60 points in total.

You have **60 minutes** to work starting from the signal to begin.

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1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Suppose $f'(x) = \sin(e^x)$ and $f(3) = 1.67$. Use the linearization of f at $x = 3$ to approximate the value of $f(\pi)$ (accurate to three decimal places).

   a. $f(\pi) \approx 1.804$

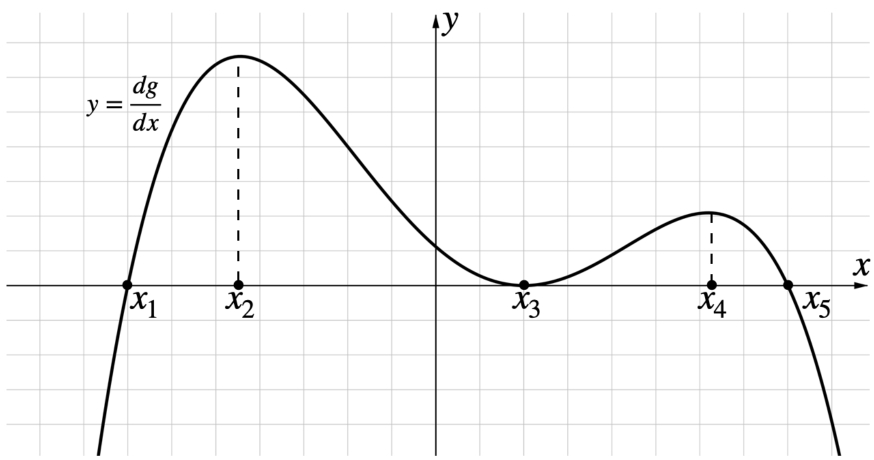
   b. $f(\pi) \approx 0.944$

   c. $f(\pi) \approx 0.134$

   d. $f(\pi) \approx 5.010$

   e. $f(\pi) \approx -0.913$

   (ii) The graph of the **derivative** of g, $y = g'(x)$, is shown below. Identify which of the following statements are true.



   I. x_2 and x_4 are critical points of g.

   II. $g(x_3)$ is a local extreme value (either maximum or minimum) of g.

   III. x_1, x_3, and x_5 are critical points of g.

   IV. $(x_2,\ g(x_2))$, $(x_3,\ g(x_3))$, and $(x_4,\ g(x_4))$ are inflection points of g.

   V. $g(x_1)$ is a local minimum of g.

   a. I only

   b. I and II only

   c. III and V only

   d. II and III only

   e. III, IV, and V only

(iii) Suppose that f and g are differentiable functions with domains $(-\infty, \infty)$. Also suppose that f is an increasing function, g is a decreasing function, and $g(x) < 0$ for all x. Which of the following can you conclude?

a. $\frac{d}{dx} f(g(x)) > 0$ for all x

b. $\frac{d}{dx} f(g(x)) < 0$ for all x

c. $\frac{d}{dx} f(g(x)) = 1$ for all x

d. There is only one value of x for which $\frac{d}{dx} f(g(x)) = 0$

e. It is impossible to determine the sign of $\frac{d}{dx} f(g(x))$ with the information provided.

(iv) Suppose f is a differentiable function such that $f(3) = -2$ and $f(7) = 6$. Which statement below must be true?

a. There exists some c in the interval $(3, 7)$ such that $f(c) = 0$.

b. There exists some c in the interval $(3, 7)$ such that $f'(c) = 0$.

c. There exists some c in the interval $(3, 7)$ such that $f(c) = 2$.

d. There exists some c in the interval $(3, 7)$ such that $f'(c) = 2$.

b. $f'(c) > 0$ for all c in the interval $(3, 7)$.

(v) Given that $f'(-2) = 0$ and $f''(x) = x(x-1)^2$. Which statement below is true? (Only one of these statements is true.)

a. f has a local minimum at $x = -2$

b. f has a local maximum at $x = -2$

c. f has a point of inflection at $x = -2$

d. f is increasing at $x = -2$

e. f is decreasing at $x = -2$

2. (*8 points*) Use calculus to determine the following for the function

$$f(x) = (xe^x)^2 .$$

   (a) (*4 points*) Find all critical points of f. Work must be shown to receive credit.

   (b) (*4 points*) Determine the absolute maximum and absolute minimum value of $f(x)$ on the interval $\left[-2, \frac{1}{2}\right]$. Work must be shown to receive credit.

3. (*4 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so**.

   (a) $\displaystyle\lim_{x\to 5} \frac{x^2-25}{5-4x-x^2} =$

   (b) $\displaystyle\lim_{x\to 0} \frac{3x^3}{\tan(x)} =$

4. (*4 points each*) Compute the following derivatives. Do not simplify.

   (a) Let $f(\theta) = \dfrac{e^{\sin(\theta)}}{\theta}$. Find $f'(\theta)$.

   (b) Let $y = 2^{x^2}$. Find the **second derivative** $\left(\frac{d^2y}{dx^2}\right)$ of y with respect to x.

5. (*6 points*) Find $\dfrac{dy}{dx}$ for the curve $e^{3x} = \sin(y) - 8y$. Solve for $\dfrac{dy}{dx}$ in terms of both x and y.

6. (*20 points*) One boat approaches a dock from the west at a constant speed of 20 km/hr. At 12:00 PM, this boat is 60 km directly west of the dock. Another boat leaves the same dock at 11:00 AM traveling directly north at a constant speed of 15 km/hr.

   (a) (*10 points*) At what rate (in units of km/hr) is the distance between the boats changing at 1:00 PM? Work must be shown to receive credit.

(b) (*10 points*) (*Refer to the same context described in Problem 6.*) What is the minimum distance between the two boats after 12:00 PM? Work must be shown to receive credit.

BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$