INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an approved calculator to

- perform operations on real numbers,
- evaluate functions at specific values, and
- generate and examine at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and, when prompted, explain your reasoning to earn full credit. For those tasks that explicitly prompt you to show work or explain your reasoning, answers alone will receive no credit. The purpose of this assessment is for you to demonstrate what you know.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 11 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 64 points in total.

You have **60 minutes** to work starting from the signal to begin.

This page is intentionally blank. You may use it for scratch work, but any work recorded on this page will not be graded.

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required. ***Record your answers in the spaces provided on page 5 of the exam.***

   (i) The average rate of change of a continuous function $y = f(x)$ over the interval $[4, 9]$ is 0.4. If $f(9) = 1$, what is the value of $f(4)$?

   a. 2

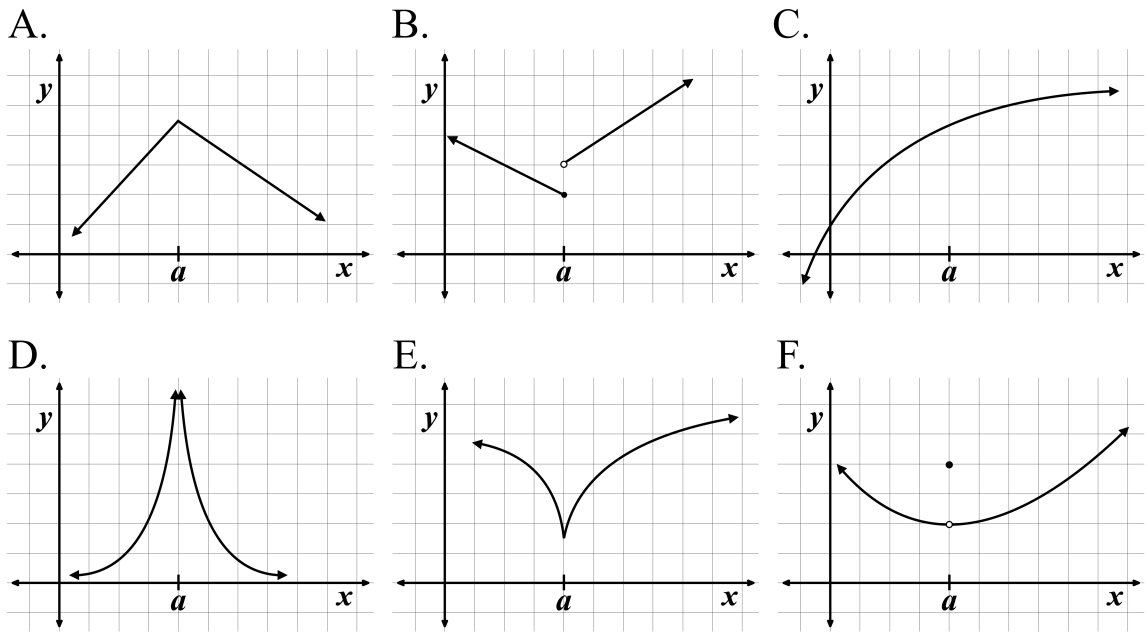
   b. 5

   c. -1

   d. -3

   e. There is not enough information to answer this question.

   (ii) Suppose $\lim_{x \to a} \dfrac{df}{dx}$ exists. Which of the following could be a graph of the function $y = f(x)$?



   a. Graphs A, C, and E only

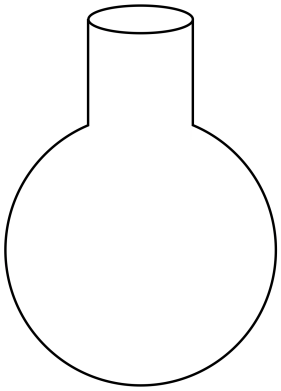
   b. Graphs C and F only

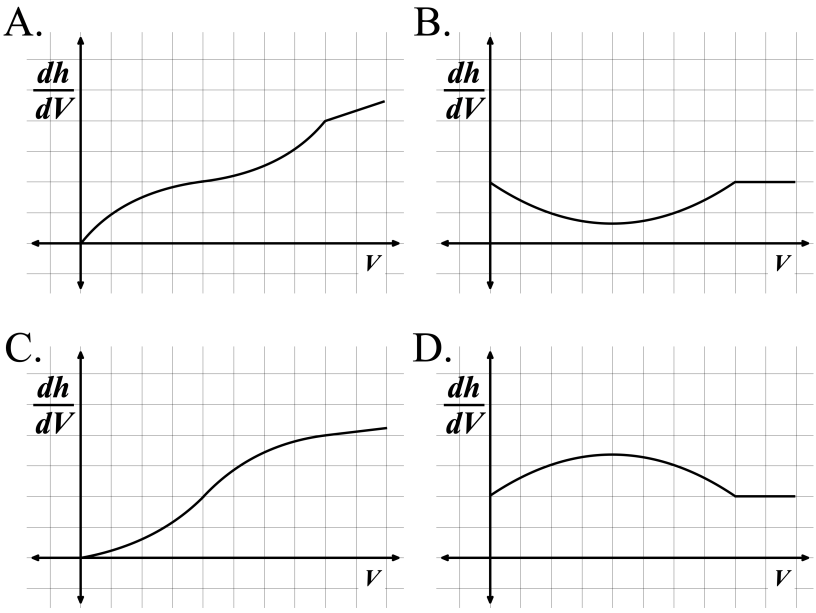
   c. Graphs B and C only

   d. Graphs A, C, D, and E only

   e. Graph C only

(iii) Imagine the vase pictured below filling with water. Let V represent the volume of water in the vase and let h represent the height of water in the vase. Which of the following graphs could represent the function $y = \dfrac{dV}{dh}$, the **derivative** of the function V with respect to h (not the function $y = V(h)$)?





a. Graph A

b. Graph B

c. Graph C

d. Graph D

e. None of these

(iv) The following expression represents the derivative of what function?

$$\lim_{\Delta x \to 0} \frac{3\cos^2(x + \Delta x) - 3\cos^2(x)}{\Delta x}$$

a. $f(x) = 3\cos^2(x + \Delta x)$

b. $f(x) = 3\cos^2(x)$

c. $f(x) = 3\cos^2(x + \Delta x) - 3\cos^2(x)$

d. $f(x) = -6\cos(x)$

e. $f(x) = \dfrac{3\cos^2(x + \Delta x) - 3\cos^2(x)}{\Delta x}$

(v) Suppose that a function $y = f(x)$ has a jump discontinuity at an input value $x = a$. Which of the following statements must be true if the function f is continuous at all other input values?

a. $f(a)$ must be undefined

b. $f(a)$ must be defined

c. $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist and are not equal

d. $\lim_{x \to a^-} f(x) = f(a)$ or $\lim_{x \to a^+} f(x) = f(a)$

e. $\lim_{x \to a^-} f(x) = f(a)$ and $\lim_{x \to a^+} f(x) = f(a)$

f. $\lim_{x \to a^-} f(x)$ exists or $\lim_{x \to a^+} f(x)$ exists, but not both

Record your answers to the five multiple choice questions in the spaces provided below. Write legibly and ***use capital letters.***

(i) [ ] (ii) [ ] (iii) [ ] (iv) [ ] (v) [ ]

2. (*3 points each*) Consider the function f defined by

$$f(x) = \begin{cases} 2x^2 - 7x - 6, & x \leq 1 \\ -5 - 8x, & x > 1 \end{cases}$$

(a) Compute $\lim_{x \to 1^-} f(x)$.

(b) Compute $\lim_{x \to 1^+} f(x)$.

(c) Does $\lim_{x \to 1} f(x)$ exist? Justify your response.

(d) Is f continuous at $x = 1$? Justify your response.

3. (*3 points each*) A car's fuel efficiency is a function of its speed. Let $F(s)$ represent the car's fuel efficiency (in gallons per mile) while driving at a speed of s miles per hour. Describe the meaning of the following expressions. Your response should identify the *quantity* represented by the expression. **Specify the units** associated with each expression.

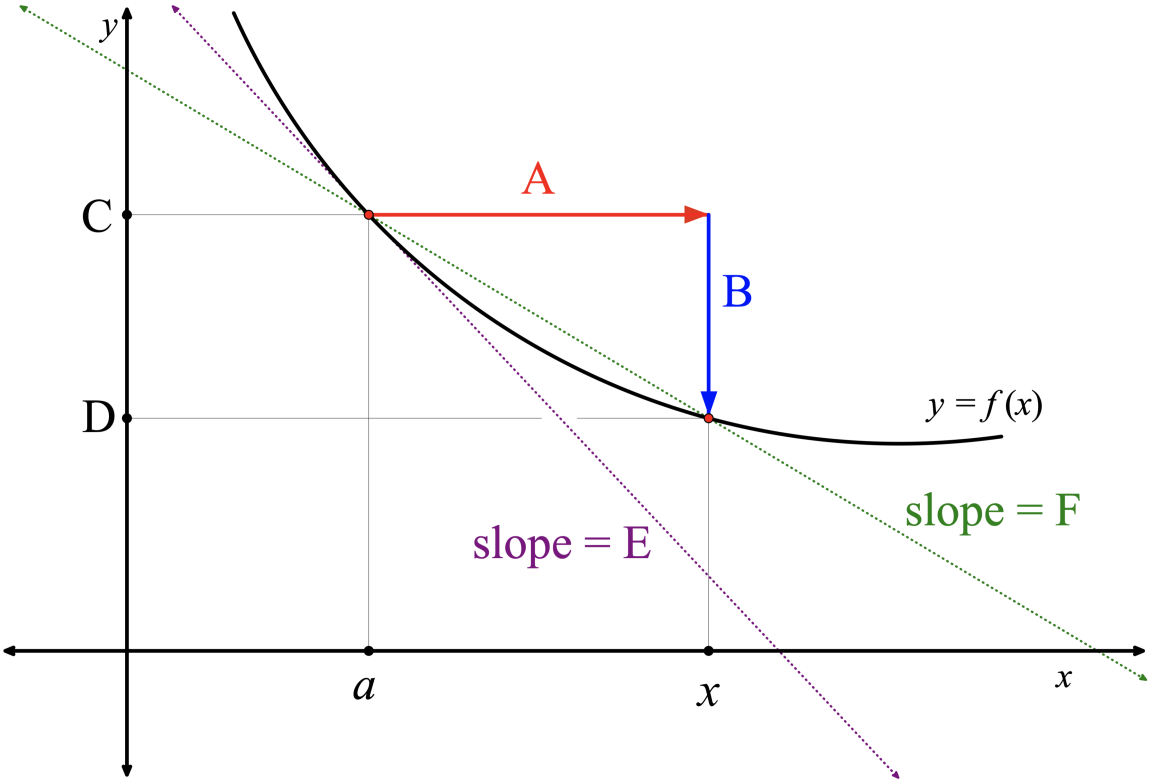
   (a) $F(50) - F(45)$.

   (b) $\dfrac{F(50) - F(45)}{50 - 45}$.

   (c) $\lim\limits_{\Delta s \to 0} \dfrac{F(70 + \Delta s) - F(70)}{\Delta s}$

   (d) What does the solution to the equation $F'(s) = 0.04$ represent?

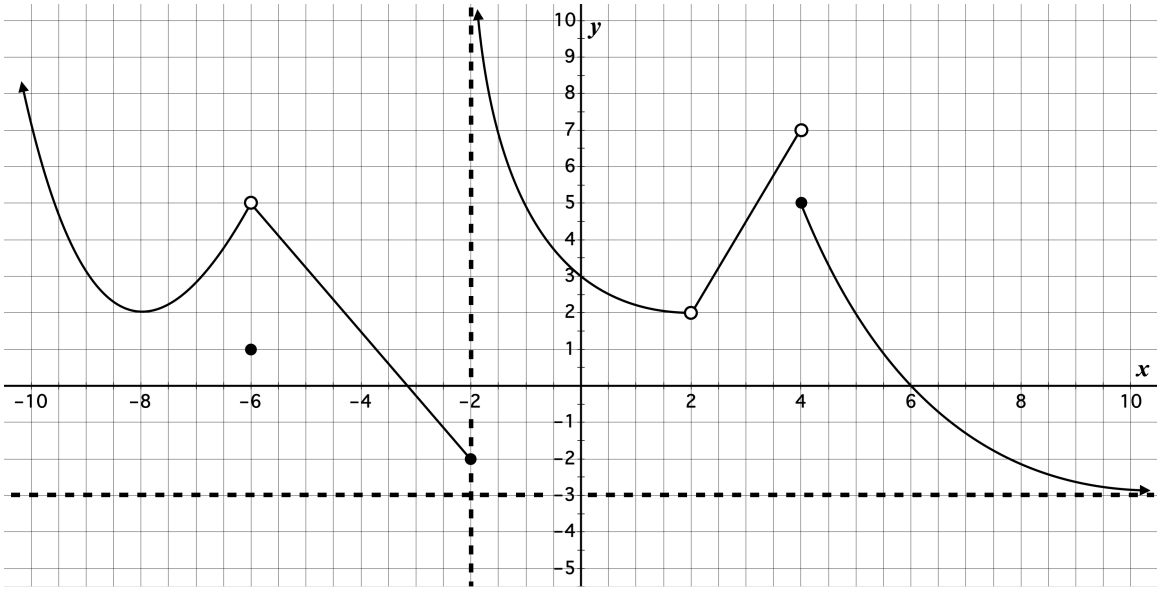
4. (*6 points*) Consider the graph of $y = f(x)$ illustrated below.



Write each graphical quantity, A-F, in the blank next to corresponding expression on the left. ***Each letter will be used exactly once.***

Expression		**Graphical Quantity**
$f(a)$	_____	A
$x - a$	_____	B
$f(x) - f(a)$	_____	C
$f(x)$	_____	D
$\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$	_____	E
$\dfrac{f(x) - f(a)}{x - a}$	_____	F

5. (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all maxima and minima, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist.

$\lim_{x \to 4^+} f(x) =$ $\quad$ $f'(-8) =$ $\quad$ $\lim_{x \to -2^-} f(x) =$

$\lim_{\Delta x \to 0} \frac{f(3+\Delta x) - f(3)}{\Delta x} =$ $\quad$ $\lim_{x \to -6} f(x) =$ $\quad$ $\left.\frac{df}{dx}\right|_{x=-5} =$

$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} =$ $\quad$ $\lim_{x \to -2^-} \frac{df}{dx} =$ $\quad$ $\lim_{x \to 2} f(x) =$

6. (*3 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate.

   (a) $\lim_{x\to 3} \dfrac{2x^2 - 18}{x - 3} =$

   6(a) answer: $\square$

   (b) $\lim_{\theta\to \frac{\pi}{2}} \dfrac{\tan(\theta)}{\sec(\theta)} =$

   6(b) answer: $\square$

   (c) Suppose f is a continuous function and that $f(2) = 7$. Evaluate the following limit.

   $\lim_{x\to 2} \dfrac{x^3 - 4f(x)}{x} =$

   6(c) answer: $\square$

7. (*6 points*) Use the limit definition of derivative to show that

$$\frac{d}{dx}\left(4 - x^2\right) = -2x.$$

You **must** use the **limit definition of derivative** to receive credit.