Math 2144 Exam 1 **Fall 2022** **NAME:**

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. Reasoning which will earn credit will use material covered in the course to date.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 8 problems on 10 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 80 points in total.

You have **60 minutes** to work starting from the signal to begin.

This page is intentionally blank. You may use it for scratch work, but any work recorded on this page will not be graded.

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Suppose that a function $y = f(x)$ has a jump discontinuity at $x = a$. Which of the following statements must be true of the function f is continuous at all other input values?

   a. $\lim_{x\to a^-} f(x) = f(a)$ and $\lim_{x\to a^+} f(x) = f(a)$

   b. $\lim_{x\to a^-} f(x) = f(a)$ or $\lim_{x\to a^+} f(x) = f(a)$

   c. $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$

   d. $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and are not equal

   e. $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

   (ii) An equation of the line tangent to the graph of $f(x) = \sqrt{x}$ at $x = 2$ is

   a. $y = \sqrt{2} + \dfrac{1}{2\sqrt{2}}(x-2)$

   b. $y = \dfrac{1}{2\sqrt{2}} + \sqrt{2}(x-2)$

   c. $y = 2 - \dfrac{1}{\sqrt{2}}\left(x - \sqrt{2}\right)$

   d. $y = 2 + \dfrac{1}{2\sqrt{x}}\left(x - \sqrt{2}\right)$

   e. $y = \sqrt{2} + \dfrac{1}{2\sqrt{x}}(x-2)$

   (iii) If $\lim_{x\to a} f(x) = L$, where L is a real number, which of the following must be true?

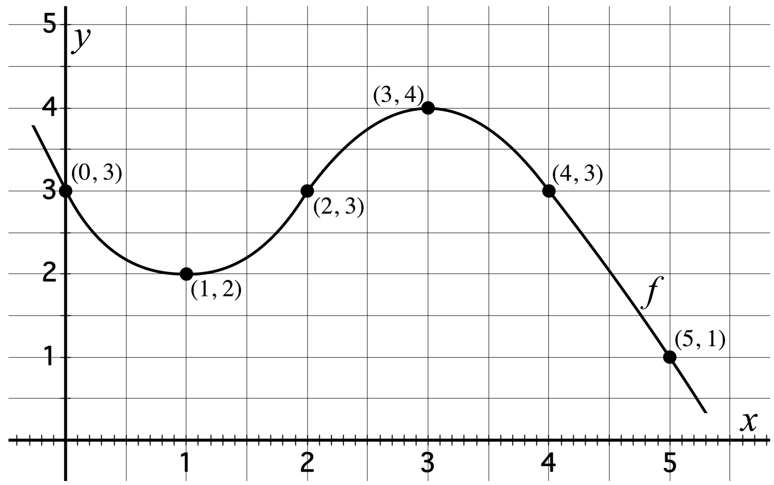
   I. $f(a) = L$

   II. $\lim_{x\to a^-} f(x) = L$

   III. $\lim_{x\to a^+} f(x) = L$

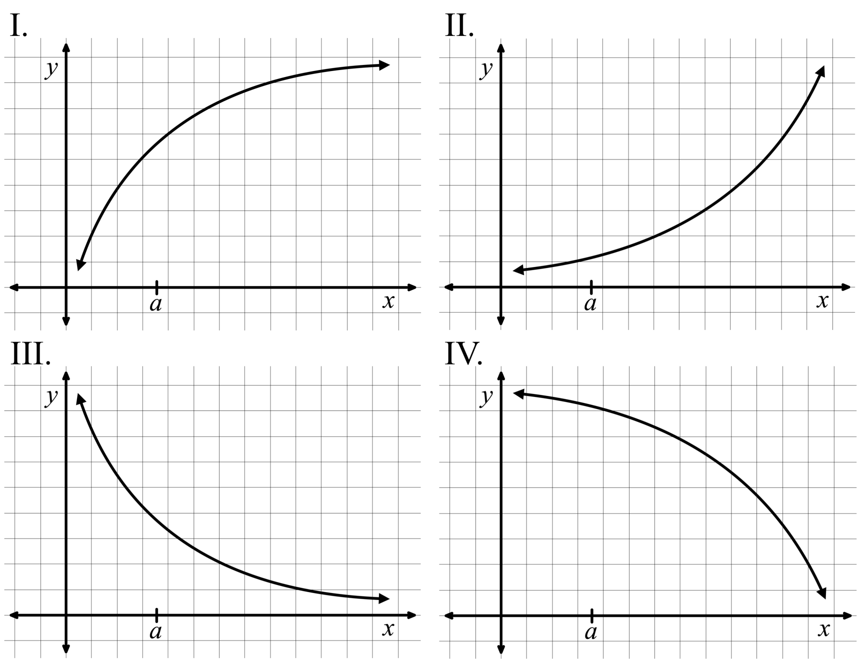
   a. I only
   b. I and II only
   c. I and III only
   d. II and III only
   e. I, II, and III

(iv) The graph of the function $y = f(x)$ is shown below. What is the average rate of change of $f(x)$ with respect to x over the interval $[0, 5]$?



a. $-\dfrac{2}{5}$ b. $-\dfrac{5}{2}$ c. $\dfrac{16}{5}$ d. $\dfrac{5}{3}$ e. $-\dfrac{5}{2}$

(v) Suppose $f'(a) < \dfrac{f(a + \Delta x) - f(a)}{\Delta x}$ for $\Delta x > 0$. Which of the following could be a graph of f?



a. I only
b. II only
c. I and IV only
d. II and III only
e. III and IV only

2. (*3 points each*) Consider the function f defined by

$$f(x) = \begin{cases} 2x^2 - 7x - 6, & x \leq 1 \\ -5 - 8x, & x > 1 \end{cases}$$

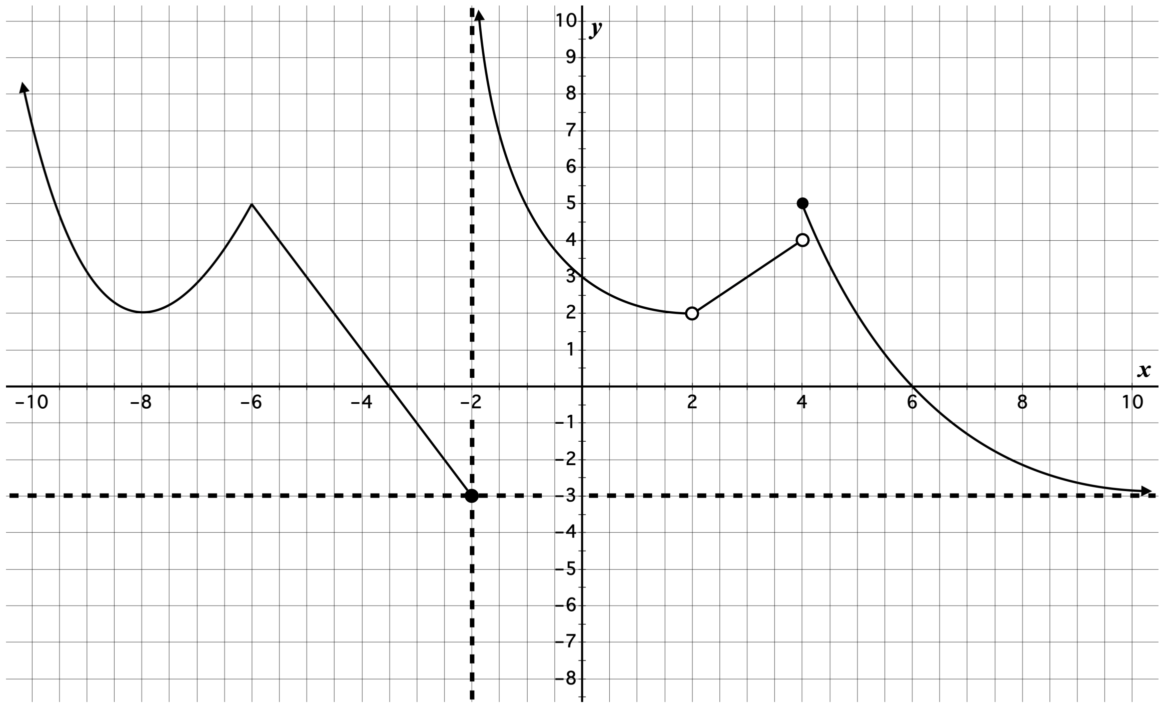
(a) Compute $\lim_{x \to 1^-} f(x)$.

(b) Compute $\lim_{x \to 1^+} f(x)$.

(c) Does $\lim_{x \to 1} f(x)$ exist? Justify your response.

(d) Is f continuous at $x = 1$? Justify your response.

3(a) (*9 points*) Answer the following questions based on the graph of f below. Assume that all critical points, points of discontinuity, and end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist.

$\lim_{x\to 2} f(x) =$

$f'(-8) =$

$\lim_{x\to -2^-} f(x) =$

$\lim_{\Delta x\to 0} \frac{f(3+\Delta x)-f(3)}{\Delta x} =$

$\lim_{x\to -6} f(x) =$

$\left.\frac{df}{dx}\right|_{x=-4} =$

$\lim_{x\to 2^+} \dfrac{f(x)-f(-6)}{x-(-6)} =$

$\lim_{x\to 0} f(x) =$

$\lim_{x\to 4} f(x) =$

3(b) (*1 point*) Identify a value of x in the interval $[-10, 10]$ at which f is continuous but not differentiable.

4. (*4 points each*) Pistol Pete is seen at a local outdoor shooting range. Forgetting that his pistols are loaded with real bullets, Pete spins his pistols and one pistol accidentally fires. When the pistol fired the bullet, it was pointing straight up. Let $d(t)$ be the height of the bullet (in feet) t seconds after firing. Express the meaning of the following **in the context of this situation** using **one or two complete sentences**.

   i. What is the practical meaning of $d(3.1) - d(3)$?

   ii. What is the practical meaning of $\dfrac{d(3+\Delta t) - d(3)}{\Delta t}$?

   iii. What is the practical meaning of the equation $d'(4) = 697$?

5. (*6 points*) Use the limit definition of derivative to show that

$$\frac{d}{dx}\left(-\frac{1}{x}\right) = \frac{1}{x^2}.$$

You **must** use the **limit definition of derivative** to receive credit. You will not earn credit for applying rules of differentiation.

6. (*5 points*) Without referring to a graph, determine whether the function

$$f(x) = \sin(x) - \ln(x)$$

has at least one real zero (or x-intercept) in the interval $(1, 3)$. Justify your response by either explaining your reasoning or citing a theorem from the text.

7. (*4 points each*) Compute the following derivatives. **Do not simplify** after taking each derivative.

   (a) Let $f(x) = 3x^4 - x^2 + 11x - 7$. Find $f'(x)$.

   (b) Let $y = x - \dfrac{8}{x^8}$. Find $\dfrac{dy}{dx}$.

   (c) Let $g(x) = (x^\pi - 1)\sqrt{x}$. Find $g'(x)$.

   (d) Let $y = \dfrac{x^8}{x^2 + 9}$. Find $\dfrac{dy}{dx}$.

8. (*3 points each*) Define the function f as follows:

$$f(x) = \frac{x^2 - c}{x - d}.$$

(a) Determine values for the constants c and d so that $y = f(x)$ has a removable discontinuity at $x = 1$. Justify your response.

(b) Determine values for the constants c and d so that the graph of $y = f(x)$ has a vertical asymptote at $x = 3$. Justify your response.

(c) Determine values for the constants c and d so that $\lim_{x \to d} f(x) = 4$. Justify your response.