MATH 2144 Calculus I
FINAL EXAM TOPICS & EXAMPLE PROBLEMS

<u>Continuity</u>

Fall 2020, Problems 7 and 11

7. Consider the function f defined by

$$f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2, & x = 1 \\ 6x - 4, & x > 1 \end{cases}$$

Which of the following are true statements about this function?

I. $\lim_{x \to 1} f(x)$ exists

II. $f(1)$ exists

III. f is continuous at $x = 1$

a. I only
b. I and II
c. II only
d. II and III
e. I, II, and III

11. Consider the function g defined by

$$g(x) = \begin{cases} x - 5, & x \le 8 \\ cx - 7, & x > 8 \end{cases}$$

Which value of c makes g continuous for all real values of x?

a. $c = -\dfrac{1}{2}$

b. $c = \dfrac{7}{5}$

c. $c = \dfrac{5}{7}$

d. $c = \dfrac{4}{5}$

e. $c = \dfrac{5}{4}$

Fall 2019, Problem 1(i)

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Consider the function f defined by

$$f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2, & x = 1 \\ 6x - 7, & x > 1 \end{cases}$$

   Which of the following are true statements about this function?

   I. $\lim_{x \to 1} f(x)$ exists

   II. $f(1)$ exists

   III. f is continuous at $x = 1$

   IV. f is differentiable at $x = 1$.

   (a) I only
   (b) I and II only
   (c) I, II, and III only
   (d) I, II, and IV only
   (e) I, II, III, and IV

Spring 2018, Problem 1(i)

1. (*3 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Consider the function f defined by

$$f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2, & x = 1 \\ 6x - 7, & x > 1 \end{cases}$$

   Which of the following are true statements about this function?

   I. $\lim_{x \to 1} f(x)$ exists

   II. $f(1)$ exists

   III. f is continuous at $x = 1$

   a. I only
   b. I and II
   c. II only
   d. II and III
   e. I, II, and III

Fall 2017, Problem 1(iv)

1. (*3 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (iv) The functions f, g, and h are defined as follows:

$$f(x) = \frac{x^2 - 1}{x - 1} \qquad g(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases} \qquad h(x) = x + 1$$

Which of the following is true?

I. $\lim_{x \to 1} g(x) = g(1)$

II. $\lim_{x \to 1} f(x) = \lim_{x \to 1} g(x) = \lim_{x \to 1} h(x)$

III. $f(1) = g(1) = h(1)$

a. I only
b. I and II
c. II only
d. II and III
e. I, II, and III

Extreme values

Fall 2020, Problems 39-42

39. If f is a differentiable function that has a local maximum at $x = c$, then which of the following is true?
    a. $f'(c) < 0$
    b. $f'(c) > 0$
    c. $f'(c) = 0$
    d. $f'(c) > f'(x)$ for all x in the domain of f'
    e. $f'(c) < f'(x)$ for all x in the domain of f'

40. Suppose $f'(c) = 0$ and $f''(x) > 0$ for all x in an open interval containing c. Which of the following is true?
    a. $f(c)$ is an inflection point
    b. $f(c)$ is a local maximum
    c. $f(c)$ is a local minimum
    d. f is increasing on an interval containing c
    e. f is decreasing on an interval containing c

41. Suppose $f'(c) = 0$ and $f''(x) > 0$ for all x around c. Which of the following is true?
    a. $f(c)$ is an inflection point.
    b. $f(c)$ is a local minimum.
    c. $f(c)$ is a local maximum.
    d. $f'(c)$ is positive.
    e. $f'(c)$ is negative.

42. Suppose $f'(c) = 0$ and $f''(x) < 0$ for all x around c. Which of the following is true?
    a. $f(c)$ is an inflection point.
    b. $f(c)$ is a local minimum.
    c. $f(c)$ is a local maximum.
    d. $f'(c)$ is positive.
    e. $f'(c)$ is negative.

Fall 2018, Problem 1(i)

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) If f is a differentiable function that has a local maximum at $x = c$, then which of the following is true?

   a. $f'(c) < 0$
   b. $f'(c) > 0$
   c. $f'(c) = 0$
   d. $f'(c) > f'(x)$ for all x in the domain of f'
   e. $f'(c) < f'(x)$ for all x in the domain of f'

<u>Linear approximation</u>

Fall 2020, Problems 51 and 52

51. Suppose that f is continuous and differentiable on the interval $[1,\ 6]$. Also suppose that $f(1) = -8$ and $f'(x) \leq 4$ for all x in the interval $[1,\ 6]$. What is the largest possible value for $f(6)$?

    a. -32
    b. -4
    c. 12
    d. 20
    e. 30

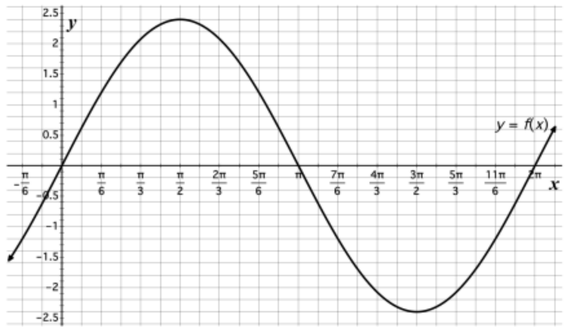
52. Suppose that f is continuous and differentiable on the interval $[3,\ 10]$. Also suppose that $f(3) = 4$ and $f'(x) \geq -2$ for all x in the interval $[3,\ 10]$. What is the smallest possible value for $f(10)$?

    a. -8
    b. 14
    c. -10
    d. -14
    e. 10

Comparing quantities graphically (e.g., derivative, concavity, average rate of change, definite integral)

Fall 2020, Problems 82-85

82. The graph of the function f is below. The following inequalities compare the values of the quantities $f'\left(\frac{\pi}{6}\right)$, $f''\left(\frac{\pi}{2}\right)$, $\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}}$, and $\int_0^{2\pi} f(x)\,dx$. Which of the following string of inequalities is true?



a. $\int_0^{2\pi} f(x)\,dx > f''\left(\frac{\pi}{2}\right) > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > f'\left(\frac{\pi}{6}\right)$

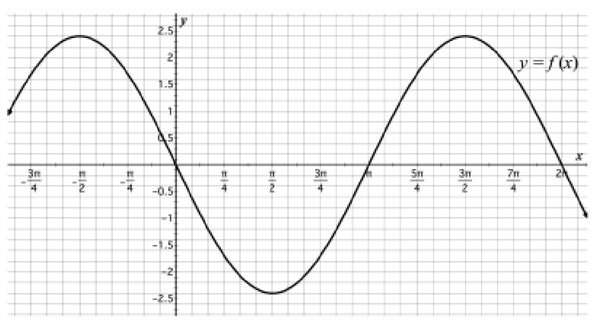
b. $f'\left(\frac{\pi}{6}\right) > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > \int_0^{2\pi} f(x)\,dx > f''\left(\frac{\pi}{2}\right)$

c. $\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > f'\left(\frac{\pi}{6}\right) > \int_0^{2\pi} f(x)\,dx > f''\left(\frac{\pi}{2}\right)$

d. $f''\left(\frac{\pi}{2}\right) > \int_0^{2\pi} f(x)\,dx > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > f'\left(\frac{\pi}{6}\right)$

e. $f'\left(\frac{\pi}{6}\right) > f''\left(\frac{\pi}{2}\right) > \int_0^{2\pi} f(x)\,dx > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}}$

83. The graph of the function f is below. The following inequalities compare the values of the quantities $f'\left(\frac{\pi}{4}\right)$, $f''(\pi)$, $\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}}$, and $\int_\pi^{2\pi} f(x)\,dx$. Which of the following string of inequalities is true?



a. $\int_\pi^{2\pi} f(x)\,dx > f''(\pi) > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}} > f'\left(\frac{\pi}{4}\right)$

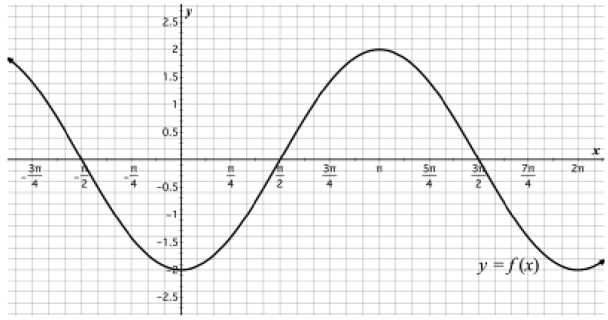
b. $f'\left(\frac{\pi}{4}\right) > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}} > \int_\pi^{2\pi} f(x)\,dx > f''(\pi)$

c. $\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}} > f'\left(\frac{\pi}{4}\right) > \int_\pi^{2\pi} f(x)\,dx > f''(\pi)$

d. $f''(\pi) > \int_\pi^{2\pi} f(x)\,dx > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}} > f'\left(\frac{\pi}{4}\right)$

e. $f'\left(\frac{\pi}{4}\right) > f''(\pi) > \int_\pi^{2\pi} f(x)\,dx > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}-\frac{\pi}{4}}$

84. The graph of the function f is below. The following inequalities compare the values of the quantities $f'\left(\frac{\pi}{2}\right)$, $f''(\pi)$, $\dfrac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$, and $\displaystyle\int_{\pi}^{2\pi} f(x)\,dx$. Which of the following string of inequalities is true?



a. $\displaystyle\int_{\pi}^{2\pi} f(x)\,dx > f''(\pi) > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right)$

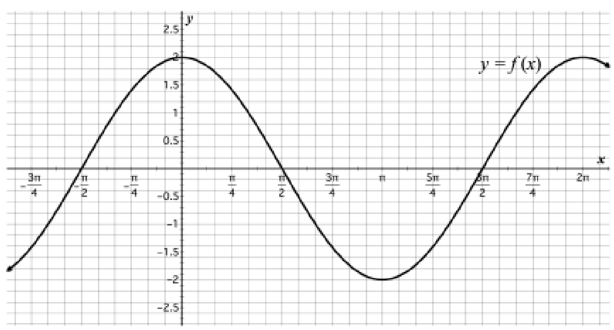
b. $\displaystyle f'\left(\frac{\pi}{2}\right) > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > \int_{\pi}^{2\pi} f(x)\,dx > f''(\pi)$

c. $\displaystyle \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right) > \int_{\pi}^{2\pi} f(x)\,dx > f''(\pi)$

d. $\displaystyle f''(\pi) > \int_{\pi}^{2\pi} f(x)\,dx > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right)$

e. $\displaystyle f'\left(\frac{\pi}{2}\right) > f''(\pi) > \int_{\pi}^{2\pi} f(x)\,dx > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$

85. The graph of the function f is below. The following inequalities compare the values of the quantities $f'\left(\frac{\pi}{2}\right)$, $f''(\pi)$, $\dfrac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$, and $\displaystyle\int_{\pi}^{2\pi} f(x)\,dx$. Which of the following string of inequalities is true?



a. $\displaystyle\int_{\pi}^{2\pi} f(x)\,dx > f''(\pi) > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right)$

b. $\displaystyle f'\left(\frac{\pi}{2}\right) > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > \int_{\pi}^{2\pi} f(x)\,dx > f''(\pi)$

c. $\displaystyle \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right) > \int_{\pi}^{2\pi} f(x)\,dx > f''(\pi)$

d. $\displaystyle f''(\pi) > \int_{\pi}^{2\pi} f(x)\,dx > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right)$

e. $\displaystyle f'\left(\frac{\pi}{2}\right) > f''(\pi) > \int_{\pi}^{2\pi} f(x)\,dx > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$

Evaluating limits (including L'Hopital's rule)

Spring 2018, Problem 3

3. (*5 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so.**

(a) $\displaystyle\lim_{x\to -3} \frac{x+3}{x^2+x-6} =$

(b) $\displaystyle\lim_{x\to 1} \frac{x^4-1}{x^3-1} =$

(c) $\displaystyle\lim_{\theta\to\frac{\pi}{2}} \frac{\cot(\theta)}{\csc(\theta)} =$

Spring 2017, Problem 3

3. (*5 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so.**

(a) $\displaystyle\lim_{x\to 8} \frac{\sqrt{x-4}-2}{x-8} =$

(b) $\displaystyle\lim_{x\to\infty} \frac{e^{2x}-1}{x^2} =$

(c) $\displaystyle\lim_{t\to -2} \frac{t^2+3t+2}{t+2} =$

(d) $\displaystyle\lim_{\theta\to\pi} \tan(\theta)\csc(\theta) =$

Fall 2017, Problem 3

3. (*5 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so.**

(a) $\displaystyle\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} =$

(b) $\displaystyle\lim_{x\to 8} \frac{x^3-64x}{x-8} =$

(c) $\displaystyle\lim_{\theta\to\frac{\pi}{2}} \frac{\tan(\theta)}{\sec(\theta)} =$

Computing derivatives

Spring 2019, Problem 5

5. (*4 points each*) Compute the following derivatives. Do not simplify.

(a) Let $f(x) = \pi^x + x^\pi$. Find $f'(x)$.

(b) Let $h(\theta) = \sec(\theta)$. Find $\dfrac{d^2h}{d\theta^2}$.

Fall 2019, Problems 5(a) and 5(b)

5. (*4 points each*) Evaluate the following. Show all of your work and give exact answers. Do not simplify.

(a) Let $y = \dfrac{\tan(x)}{x^2}$. Find $\dfrac{dy}{dx}$.

(b) Let $g(t) = \sin^{-1}\left(\sqrt{t}\right)$. Find $g'(t)$.

Spring 2018, Problem 5

5. (*5 points each*) Compute the following derivatives. Do not simplify.

(a) Let $y = \cos^2(3x)$. Find $\dfrac{dy}{dx}$.

(b) Let $f(t) = 4e^{-t} + 5e^t$. Find $f^{(4)}(t)$.

(c) Let $g(x) = \ln(x)\sec(x)$. Find $g'(x)$.

Fall 2018, Problem 5

5. (*4 points each*) Compute the following derivatives. Do not simplify.

(a) Let $y = \dfrac{\sin(x)}{4x}$. Find $\dfrac{dy}{dx}$.

(b) Let $f(t) = \sqrt{t^2 + 3}$. Find $f'(t)$.

(c) Let $g(x) = \tan^{-1}(\cos(x))$. Find $g'(x)$.

Spring 2017, Problem 6

6. (*5 points each*) Compute the following derivatives. Do not simplify.

(a) Let $y = \dfrac{2^x}{x^3 - 1}$. Find $\dfrac{dy}{dx}$.

(b) Let $f(t) = t\sin(3t)$. Find $\dfrac{df}{dt}$.

(c) Let $g(x) = \ln(x^2)$. Find $g''(x)$.

(d) Find $\dfrac{dy}{dx}$ for the curve $\sin(y) = x + \cos(y)$. (You must solve for $\dfrac{dy}{dx}$ in terms of x and y).

Fall 2017, Problem 8

8. (*5 points each*) Compute the following derivatives. Do not simplify.

(a) Let $y = \dfrac{x\sin(x)}{e^x}$. Find $\dfrac{dy}{dx}$.

(b) Let $f(x) = \tan^3(x) + \tan(x^3)$. Find $\dfrac{df}{dx}$.

(c) Let $g(t) = 4^t$. Find $g''(t)$.

Evaluating integrals

Spring 2019, Problem 6

6. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) $\displaystyle\int (3e^x + 2\sin(x))\ dx =$

(b) $\displaystyle\int_{-3}^{1} 2x\sqrt{x^2+1}\, dx =$

Fall 2019, Problems 5(c) and 5(d)

5. (*4 points each*) Evaluate the following. Show all of your work and give exact answers. Do not simplify.

(c) $\displaystyle\int_{\pi}^{0} (2+\cos(\theta))\ d\theta =$

(d) $\displaystyle\int (3x-4)(3x^2-8x+6)^7\, dx =$

Spring 2018, Problem 9

9. (*5 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) $\displaystyle\int \frac{x}{\sqrt{x^2+5}}\, dx =$

(b) $\displaystyle\int_{-2}^{3} e^{5x}\, dx =$

(c) $\displaystyle\frac{d}{dx}\int_{1}^{x^2} (t+2)\, dt =$

Fall 2018, Problem 9

9. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) $\displaystyle\int 3\sec^2(-8x)\,dx =$

(b) $\displaystyle\int_1^4 \frac{2}{t^2}\,dt =$

(c) $\displaystyle\frac{d}{dx}\int_\pi^x \frac{\sin(\theta)}{\theta}\,d\theta =$

Spring 2017, Problem 9

9. (*5 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) $\displaystyle\int_0^\pi \cos(\theta)3^{\sin(\theta)}d\theta =$

(b) $\displaystyle\int \left(x+\sec^2(x)\right)dx =$

(c) $\displaystyle\frac{d}{dx}\int_1^x \left(t^5-7t^3\right)dt =$

(d) $\displaystyle\int \frac{\ln(x)}{x}dx =$

Fall 2017, Problem 10

10. (*5 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) $\displaystyle\int \left(4x^7-\sin(x)\right)dx =$

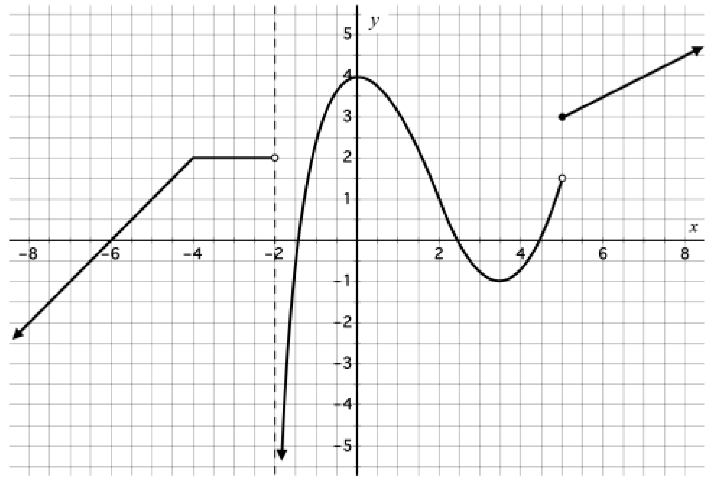
(b) $\displaystyle\int \frac{x}{(x^2+1)^3}\,dx =$

(c) $\displaystyle\frac{d}{d\theta}\int_{-2}^{\cos(\theta)} x^4\,dx =$

Evaluating expressions graphically

Spring 2019, Problem 2(a)

2(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, points of inflection, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.

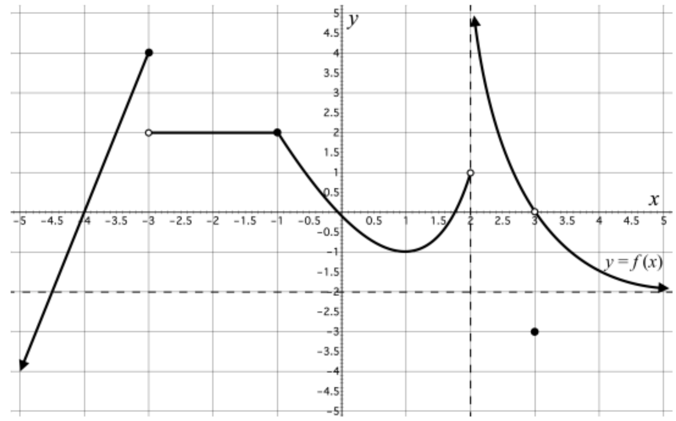


Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

$\lim_{x \to -4} f(x) =$	$\left.\frac{d^2 f}{dx^2}\right\|_{x=6} =$	$\lim_{x \to 5^-} f(x) =$
$\int_{-8}^{-4} f(x)\, dx =$	$f'(-5.7) =$	$\lim_{\Delta x \to 0} \frac{f(-3+\Delta x)-f(-3)}{\Delta x} =$
$\frac{f(6.8) - f(5.1)}{6.8 - 5.1} =$	$\lim_{x \to -2^+} f(x) =$	$f'(0) =$

Fall 2019, Problem 2(a)

2(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

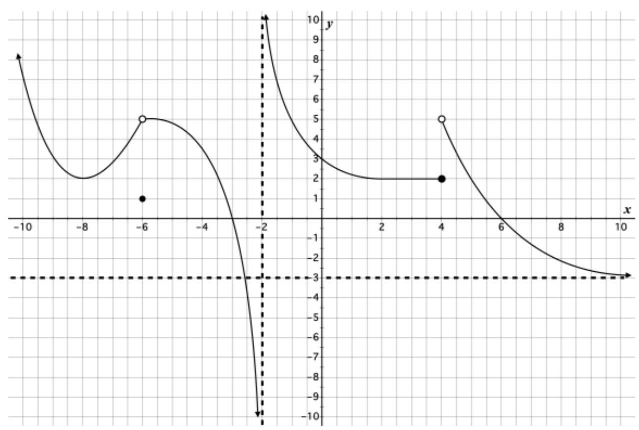
$\lim_{x\to -3} f(x) =$ $\quad\quad$ $\left.\frac{d^2 f}{dx^2}\right|_{x=-4.7} =$ $\quad\quad$ $\int_{-4}^{-1} f(x)\,dx =$

$f'(-3.4) =$ $\quad\quad$ $\lim_{x\to 2^-} f(x) =$ $\quad\quad$ $\lim_{\Delta x\to 0} \frac{f(1+\Delta x)-f(1)}{\Delta x} =$

$f'(-1) =$ $\quad\quad$ $\frac{d}{dx}\int_{-2}^{1} f(t)\,dt =$ $\quad\quad$ $\lim_{N\to\infty} \sum_{k=1}^{N} f\left(-2+\frac{k}{N}\right)\cdot\frac{1}{N} =$

Spring 2018, Problem 2

2. (*9 points*) Answer the following questions based on the graph of f below. Assume that all critical points, points of discontinuity, and points of inflection of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

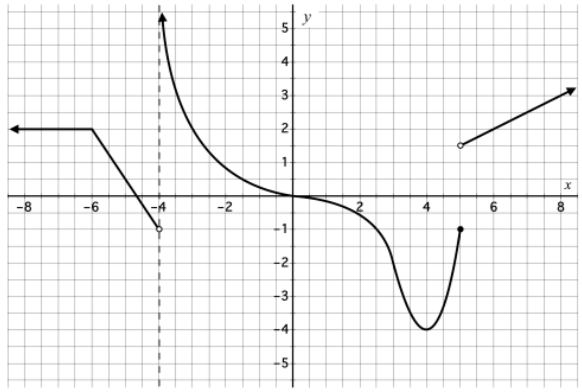
$\lim_{x\to -6} f(x) =$ $\quad\quad$ $f'(-8) =$ $\quad\quad$ $\lim_{x\to -2^+} f(x) =$

$\int_{2}^{4} f(x)\,dx =$ $\quad\quad$ $\lim_{x\to 4} f(x) =$ $\quad\quad$ $\lim_{\Delta x\to 0} \frac{f(3+\Delta x)-f(3)}{\Delta x} =$

$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} =$ $\quad\quad$ $\lim_{x\to\infty} f(x) =$ $\quad\quad$ $\lim_{x\to 0} f(x) =$

Fall 2018, Problem 2(a)

2(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, points of inflection, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

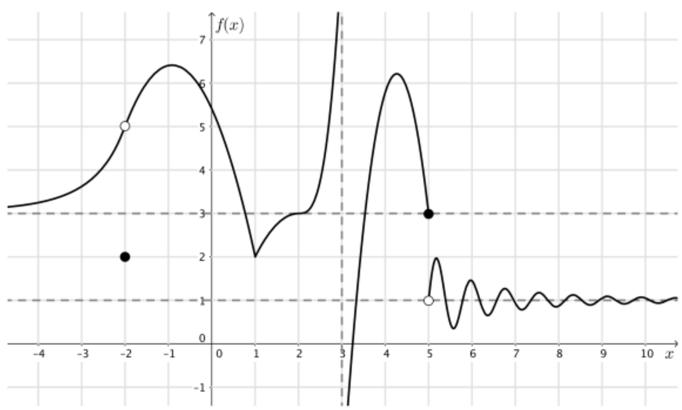
$\lim_{x\to -4^+} f(x) =$ $\qquad$ $f'(8) =$ $\qquad$ $\lim_{x\to 5^-} f(x) =$

$\int_{-8}^{-6} f(x)\,dx =$ $\qquad$ $\left.\frac{d^2 f}{dx^2}\right|_{x=-5} =$ $\qquad$ $\lim_{\Delta x\to 0} \frac{f(4+\Delta x) - f(4)}{\Delta x} =$

$\frac{f(7.3) - f(6)}{7.3 - 6} =$ $\qquad$ $\lim_{x\to -\infty} f(x) =$ $\qquad$ $f''(0) =$

Spring 2017, Problem 2

2. (*9 points*) Answer the following questions based on the graph of f below. Assume that all critical points, point of discontinuity, and points of inflection of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

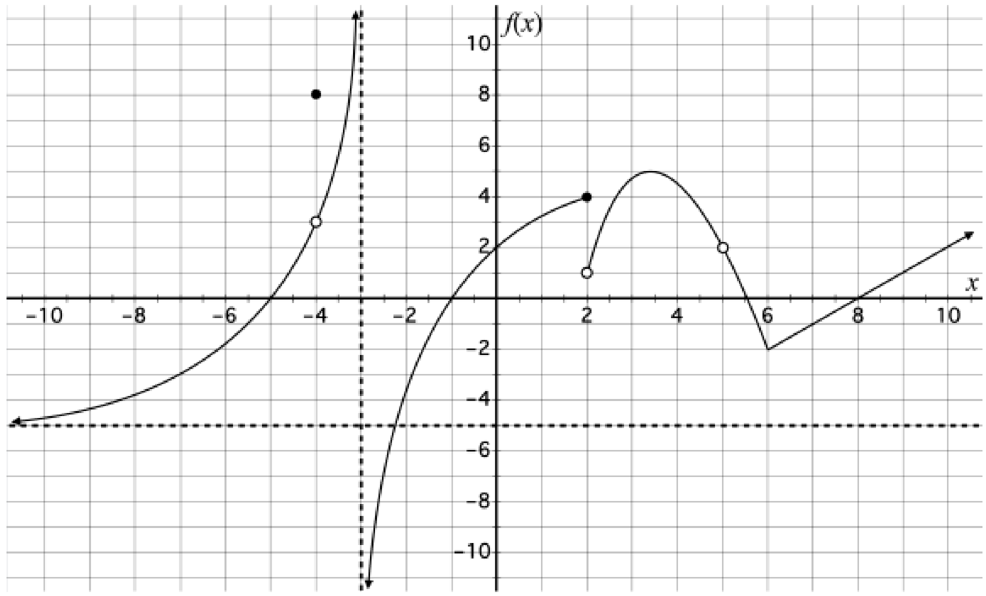
$f(-2) =$ $\qquad$ $f(5) =$ $\qquad$ $\lim_{x\to -2} f(x) =$

$\lim_{x\to -\infty} f(x) =$ $\qquad$ $\lim_{x\to \infty} f(x) =$ $\qquad$ $f''(2) =$

$\lim_{x\to 3^-} f(x) =$ $\qquad$ $\lim_{x\to 5^+} f(x) =$ $\qquad$ $f'(1) =$

Fall 2017, Problem 2

2. (*9 points*) Answer the following questions based on the graph of f below. Assume that all critical points, points of discontinuity, and points of inflection of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

$f(-4) =$

$f'(8) =$

$\lim_{x \to -3^+} f(x) =$

$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{h \to 0} \dfrac{f(9+h) - f(9)}{h} =$

$\lim_{x \to 5} f(x) =$

$\lim_{x \to \infty} f(x) =$

$f'(6) =$

Interpreting average and instantaneous rates of change in context

Fall 2020, Problems 19-23

19. The function $y = g(t)$ measures the amount of iron in Mikayla's bloodstream (in milligrams) where t is measured in minutes since she ingested an iron supplement. What is the most appropriate interpretation of $g'(28) = 3.7$?

    a. Twenty-eight minutes after ingesting the iron supplement, the amount of iron in Mikayla's bloodstream was growing at a rate of 3.7 milligrams per minute.

    b. Twenty-eight minutes after ingesting the iron supplement, Mikayla had 3.7 milligrams of iron in her bloodstream.

    c. The amount of iron in Mikayla's bloodstream grew by 3.7 milligrams during the 29th minute after she ingested the iron supplement.

    d. On average, the amount of iron in Mikayla's bloodstream increased by 3.7 milligrams each minute over the first 28 minutes since she ingested the iron supplement.

    e. The amount of iron in Mikayla's bloodstream grew by 3.7 milligrams during the first 28 minutes after she ingested the iron supplement.

20. The function $y = f(x)$ measures the U.S. production of natural gas (in millions of cubic feet) at time x where x is measured in years since January 1, 1970. What is the most appropriate interpretation of $f'(35) = 579$?

    a. On January 1, 2005, U.S. natural gas production was growing at a rate of 579 million cubic feet per year.

    b. The U.S. produced 579 million more cubic feet of natural gas in 2005 than 1970.

    c. U.S. natural gas production was 579 million cubic feet in 2005.

    d. On average, U.S. natural gas production increased by 579 million cubic feet per year over the first 35 years following 1970.

    e. U.S. natural gas production grew by 579 million cubic feet from 2004 to 2005.

21. Courtney is driving from Stillwater to Denver. The function $y = f(t)$ measures the amount of fuel Courtney's car has consumed (in gallons) where t is measured in hours since Courtney left Stillwater. What is the most appropriate interpretation of $f'(5) = 2.5$?

    a. On average, the amount of fuel Courtney's car consumed increased by 2.5 gallons each hour over the first 5 hours since she left Stillwater.

    b. The amount of fuel Courtney's car has consumed increased by 2.5 gallons during the 5th hour after she left Stillwater.

    c. Courtney's car had 2.5 gallons of fuel in its tank 5 hours after leaving Stillwater.

    d. Five hours after leaving Stillwater, the amount of fuel Courtney's car was consuming was changing at a rate of 2.5 gallons per hour.

    e. Courtney's car consumed 2.5 gallons of fuel during the first 5 hours of her drive from Stillwater to Denver.

22. The temperature T(in degrees Fahrenheit) of a 12 ounce cup of coffee at time t (in minutes since the coffee was brewed) is given by $T(t) = \frac{3}{8}t^2 - 13t + 180$ for $0 \leq t \leq 10$. What is the most appropriate interpretation of the statement

$$T'(4.2) = -9.85?$$

a. 4.2 minutes after the coffee was brewed, the temperature of the coffee was 9.85 degrees Fahrenheit less than the initial brew temperature.

b. The temperature of the coffee decreased by 9.85 degrees Fahrenheit during the fourth minute after the coffee was brewed.

c. On average, the temperature of the coffee decreased by 9.85 degrees Fahrenheit each minute over the first 4.2 minutes since it was brewed.

d. 4.2 minutes after the coffee was brewed, the temperature of the coffee was changing at a rate of -9.85 degrees Fahrenheit per minute.

e. The temperature of the coffee was approximately 132.01 degrees Fahrenheit exactly 4.2 minutes after the coffee was brewed.

23. The function f gives the amount of water (in thousands of gallons) in a Stillwater water tower t hours after noon on September 17, 2019. Which of the following statements best describes the meaning of $f'(6.5) = -117$?

a. At exactly 6:30 PM on September 17, 2019, water was draining from the tower at an instantaneous rate of 117,000 gallons per hour.

b. Exactly 117,000 gallons of water drained from the water tower from noon to 6:30 PM on September 17, 2019.

c. Water was draining from the tower at an instantaneous rate of 117,000 gallons per hour from noon to 6:30 PM on September 17, 2019.

d. The volume of water in the tower is 117,000 gallons at exactly 6:30 PM on September 17, 2019.

e. The average of the instantaneous rates at which water was draining from the tower from noon to 6:30 PM on September 17, 2019 is 117,000 gallons per hour.

Interpreting Riemann sums in context

Fall 2020, Problems 53-58, 60, 61, 62, 63, 64, 67, 70

53. The expression $\frac{1}{30}\left(\sin\left(\frac{1}{30}\right)+\sin\left(\frac{2}{30}\right)+\sin\left(\frac{3}{30}\right)+\cdots+\sin\left(\frac{30}{30}\right)\right)$ is a Riemann sum approximation for which of the following expressions?

a. $\int_0^1 \sin\left(\frac{x}{30}\right)dx$

b. $\int_0^1 \sin(x)dx$

c. $\frac{1}{30}\int_0^1 \sin\left(\frac{x}{30}\right)dx$

d. $\frac{1}{30}\int_0^1 \sin(x)dx$

e. $\frac{1}{30}\int_0^{30} \sin(x)dx$

54. The expression $\frac{1}{10}\left(\left(\frac{1}{10}\right)^2+\left(\frac{2}{10}\right)^2+\left(\frac{3}{10}\right)^2+\cdots+\left(\frac{20}{10}\right)^2\right)$ is a Riemann sum approximation for which of the following expressions?

a. $\int_0^2 x^2dx$

b. $\int_0^2 \left(\frac{x}{10}\right)^2 dx$

c. $\frac{1}{10}\int_0^2 \left(\frac{x}{10}\right)^2 dx$

d. $\int_0^{20} x^2dx$

e. $\frac{1}{10}\int_0^2 x^2dx$

55. The expression $\frac{1}{25}\left(\cos\left(1+\frac{1}{25}\right)+\cos\left(1+\frac{2}{25}\right)+\cos\left(1+\frac{3}{25}\right)+\cdots+\cos\left(1+\frac{50}{25}\right)\right)$ is a Riemann sum approximation for which of the following expressions?

a. $\int_1^3 \cos(x)\,dx$

b. $\frac{1}{25}\int_{25}^{50} \cos(x)\,dx$

c. $\int_1^3 \cos\left(1+\frac{x}{25}\right)dx$

d. $\frac{1}{25}\int_0^1 \cos\left(1+\frac{x}{25}\right)dx$

e. $\int_1^{50} \cos(x)\,dx$

56. The expression $\sum_{k=1}^{10}\left(1+\frac{3k}{10}\right)^3 \cdot \frac{3}{10}$ is a Riemann sum approximation for which of the following?

a. $\frac{3}{10}\int_1^{10} x^3\,dx$

b. $\int_1^{10} (1+x)^3\,dx$

c. $\frac{3}{10}\int_1^{10}\left(1+\frac{3x}{10}\right)^3\,dx$

d. $\int_1^{4} x^3\,dx$

e. $\frac{3}{10}\int_1^{4} x^3\,dx$

57. The expression $\sum_{k=1}^{6}\sin(2+0.5k)\cdot 0.5$ is a Riemann sum approximation for which of the following integrals?

a. $0.5\int_1^{6}\sin(2+0.5x)\,dx$

b. $\int_2^{5}\sin(x)\,dx$

c. $0.5\int_1^{6}\sin(2+x)\,dx$

d. $\int_1^{6}\sin(x)\,dx$

e. $0.5\int_2^{5}\sin(x)\,dx$

58. The function $y = g(t)$ represents the relationship between the rate of change in the value of investment stocks (in dollars per month) and the number of months t elapsed since the stocks were purchased. Which of the following sums approximates the change in the value of the stocks over the interval of time from 4 to 7 months after the stocks were purchased?

a. $\sum_{k=4}^{7} g(k)$

b. $\sum_{k=4}^{7} g(t)\cdot\Delta t$

c. $\sum_{k=1}^{6} g(4+0.5k)\cdot 0.5$

d. $\sum_{k=0}^{3} g(4+k)\cdot\Delta t$

e. $\sum_{k=0}^{3} g(4+k)$

60. Oil leaks out of a tank at a rate of $r = f(t)$ gallons per minute, where t is measured in minutes. Which of the following expressions represents the **exact** amount of oil that leaked out of the tank from 15 to 45 minutes after oil started leaking?

I. $f(45) - f(15)$

II. $\displaystyle\int_{15}^{45} f(t)\,dt$

III. $\displaystyle\sum_{k=1}^{60} f\left(15 + \frac{k}{2}\right) \cdot \frac{1}{2}$

IV. $\displaystyle\lim_{N\to\infty} \sum_{k=1}^{N} f\left(15 + \frac{30k}{N}\right) \cdot \frac{30}{N}$

a. I only
b. I and II only
c. II and IV only
d. I, II, and IV only
e. I, II, III, and IV

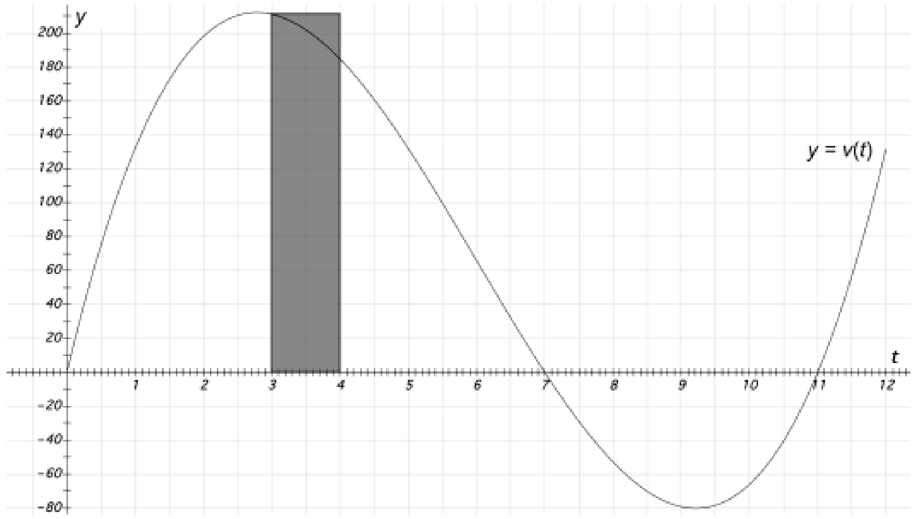
61. The function $g(t)$ gives the rate at which oil leaves a tanker, and is decreasing between 2 minutes and 10 minutes. Which of the following are underestimates of the amount of oil that left the tank between 2 and 10 minutes.

a. $g(2) + g(3) + g(4) + g(5) + g(6) + g(7) + g(8) + g(9)$
b. $\displaystyle\sum_{k=3}^{10} g(k)$
c. $\displaystyle\sum_{j=1}^{20} g\left(2 + \frac{j-1}{2}\right) \cdot \frac{1}{2}$
d. $\displaystyle\sum_{j=1}^{20} g\left(2 + \frac{j}{2}\right) \cdot \frac{1}{2}$
e. $2 \cdot (g(4) + g(6) + g(8) + g(10))$
f. $2 \cdot (g(3) + g(5) + g(7) + g(9))$

62. Kacey decides to go for a run before school. She starts her run from home. The function $y = v(t)$ expresses the relationship between Kacey's velocity (in meters per minute) as she runs and the number of minutes elapsed since she started running. What quantity does the sum $\displaystyle\sum_{k=1}^{6} v\left(1 + \frac{k-1}{2}\right) \cdot \frac{1}{2}$ approximate?

a. The average rate of change of Kacey's velocity over the interval of time from $t = 1$ to $t = \frac{7}{2}$.
b. The change in Kacey's distance away from home over the interval of time from $t = 1$ to $t = 4$.
c. Kacey's acceleration over the interval of time from $t = 1$ to $t = 4$.
d. Kacey's distance away from home after having run for 3.5 minutes.
e. Kacey's instantaneous velocity 3.5 minutes after having left home.

63. Yvonne decides to go for a run before school. She starts her run from home. The function $y = v(t)$ expresses the relationship between Yvonne's velocity (in meters per minute) as she runs and the number of minutes t elapsed since she started running. What quantity does the area of the rectangle on the graph of the function $y = v(t)$ below approximate?



a. Yvonne's average velocity over the interval of time from $t = 3$ to $t = 4$.
b. Yvonne's distance away from home 3 minutes after she started running.
c. The velocity at which Yvonne runs over the interval of time from $t = 3$ to $t = 4$.
d. Yvonne's acceleration over the interval of time from $t = 3$ to $t = 4$.
e. The change in Yvonne's distance away from home over the interval of time from $t = 3$ to $t = 4$.

64. Yvonne decides to go for a run before school. She starts her run from home. The function $y = v(t)$ expresses the relationship between Yvonne's velocity (in meters per minute) as she runs and the number of minutes t elapsed since she started running. What quantity does the sum $v(1) \cdot \frac{1}{2} + v\left(\frac{3}{2}\right) \cdot \frac{1}{2} + v(2) \cdot \frac{1}{2} + v\left(\frac{5}{2}\right) \cdot \frac{1}{2} + v(3) \cdot \frac{1}{2} + v\left(\frac{7}{2}\right) \cdot \frac{1}{2}$ approximate?

a. The average rate of change of Yvonne's velocity over the interval of time from $t = 1$ to $t = \frac{7}{2}$.
b. The change in Yvonne's distance away from home over the interval of time from $t = 1$ to $t = 4$.
c. Yvonne's acceleration over the interval of time from $t = 1$ to $t = 4$.
d. Yvonne's distance away from home after having run for 3.5 minutes.
e. Yvonne's instantaneous velocity 3.5 minutes after having left home.

67. A tank contains 10 gallons of water initially. Water is then poured into the tank at a rate of $r(t)$ gallons per minute. Which of the following equations give the exact amount of water in the tank after 5 minutes?

a. $\displaystyle\sum_{k=1}^{10} r(\frac{k}{2}) \cdot \frac{1}{2}$

b. $\displaystyle\int_0^5 r(t)\ dt + 10$

c. $\displaystyle\int_0^5 r(t)\ dt$

d. $r(5) - r(0)$

70. Golf balls are made in a factory at a rate of $r(t)$ balls per week. Let $A(t)$ be the function representing the amount of golf balls that have been constructed up to a time t. Which of the following represent the exact amount of golf balls produced in this factory during the first four weeks of production?

I. $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} r(\frac{k}{4n}) \cdot \frac{k}{4n}$

II. $\displaystyle\int_0^4 r(t)\ dt$

III. $\displaystyle\sum_{k=1}^{10} r(\frac{k}{40}) \cdot \frac{k}{40}$

a. I only
b. II only
c. I and II only
d. I and III only
e. I, II, and III

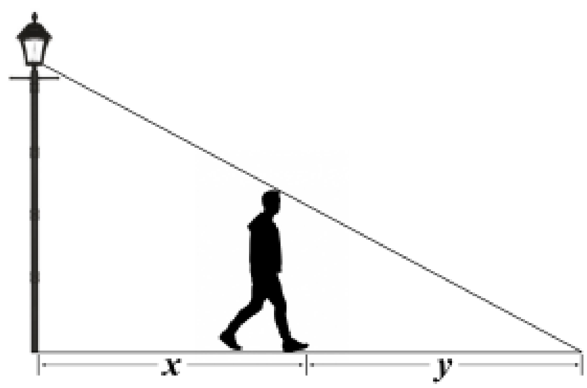
Related Rates

Spring 2019, Problem 8

8. (*6 points*) A cylindrical cup with radius 1.5 inches is filling with water at a constant rate of 2 cubic inches per second. At what constant rate is the water level rising?

Spring 2018, Problem 7

7. (*8 points*) A 6 foot-tall man is walking away from a lamp post, which is 11 feet tall. Let x represent the man's distance from the lamp post (in feet) and let y represent the length of the man's shadow (in feet), as shown in the picture below. Determine how fast the man is walking if the length of his shadow is increasing at a rate of 12 feet per second.



Fall 2018, Problem 7

7. (*6 points*) The volume of a sphere is given by the equation $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere (in meters). Suppose r is increasing at a rate of 3 meters per second. Determine the rate at which the volume is increasing when $r = 2$ meters.

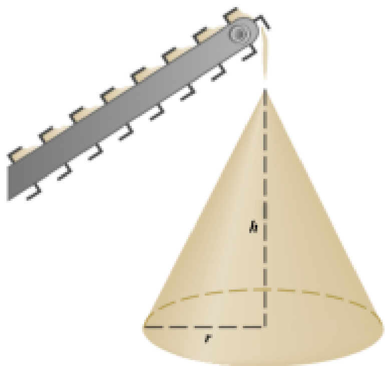
Spring 2017, Problem 7

7. (*10 points*) An airplane is flying at an altitude of 8 kilometers and passes directly over a radar antenna. When the distance between the plane and the antenna is 12 kilometers, the radar detects that the distance between the plane and the antenna is changing at a rate of 340 kilometers per hour. What is the speed of the airplane at that moment? (Round your answer to three decimal places.)

Fall 2017, Problem 7

7. (*10 points*) Suppose gravel is being poured into a conical pile at a rate of 5 m^3/s, and suppose that the radius r of this cone is always half its height h. How fast is the height of the pile increasing when the height is 10 m?

(*Note that the formula for the volume of a cone is* $V = \frac{1}{3}\pi r^2 h$).



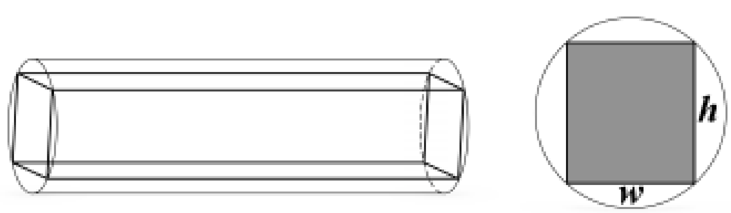
Optimization

Spring 2019, Problem 9

9. (*10 points*) You have 100 feet of fence to make a rectangular garden alongside the exterior wall of your house. (The wall of the house bounds one side of the garden). What is the largest area possible (in square feet) for the garden?

Fall 2019, Problem 8

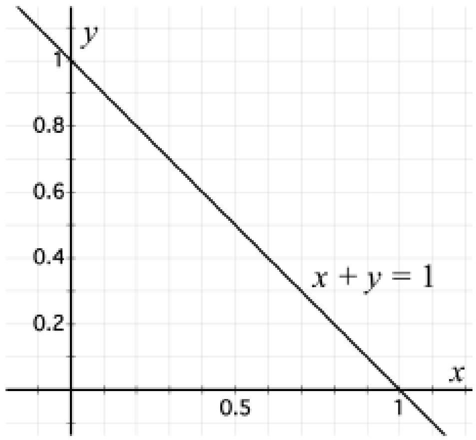
8. (*10 points*) Suppose a rectangular beam is cut from a cylindrical log of diameter 40 cm as shown in the image below. The strength of a beam, S, is given by the formula $S = 7wh^2$, where w represents the width of the beam in centimeters and h represents the height of the beam in centimeters. Find the width and height of the beam with maximum strength that can be cut from the log. Justify your answer.



Cross-section

Spring 2018, Problem 8

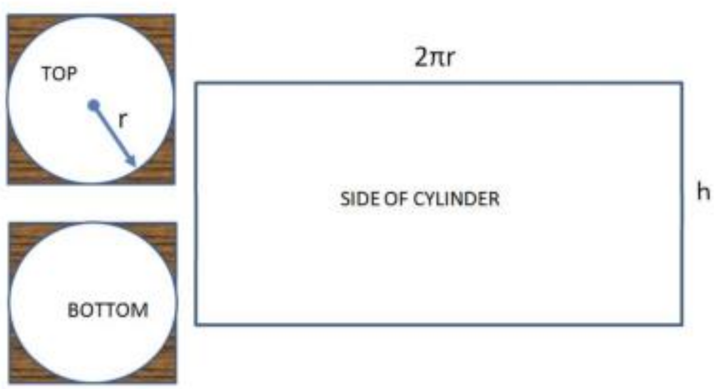
8. (*8 points*) **Use calculus** to find the dimensions of the rectangle of largest area that can be inscribed in the region bounded by $x + y = 1$, and the positive x and y axes.



Fall 2018, Problem 8

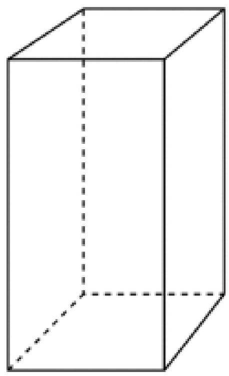
8. (*10 points*) The top and bottom of a cylinder will be constructed from square pieces of cardboard (with the excess material thrown away). The side (i.e., lateral surface) of the cylinder will be made of a rectangular piece of cardboard. Altogether, 2,400 square inches of cardboard will be used (including the wasted amount). What is the radius of the cylinder of largest volume that can be constructed in this way? **Justify your response.**

(*Recall that the volume of a cylinder is given by* $V = \pi r^2 h$.)



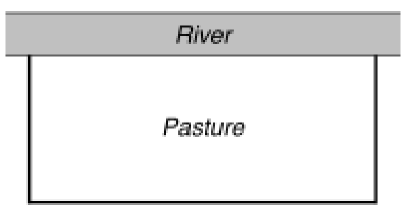
Spring 2017, Problem 8

8. (*10 points*) We want to construct a rectangular box with a square base as shown below. The box will have a surface area of 12 square feet. Use calculus to determine the maximum volume of the box. Write your solution as an exact value, not a decimal approximation.



Fall 2017, Problem 9

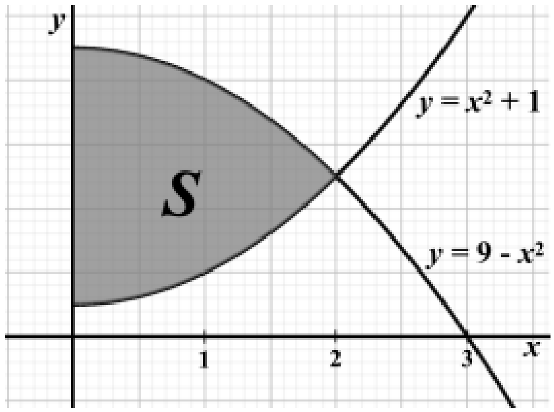
9. (*10 points*) A rectangular pasture is being created with one side along a straight riverbank. The remaining three sides are to be enclosed with a fence. If there are 12 km of fence available, use calculus to find the dimensions of the rectangular pasture to maximize its area.



Area between two curves

Spring 2019, Problem 10(a)

10(a) (*6 points*) Let S be the region bounded by the graphs of $y = x^2 + 1$ and $y = 9 - x^2$ as shown in the graph below. Find the area of the region S.



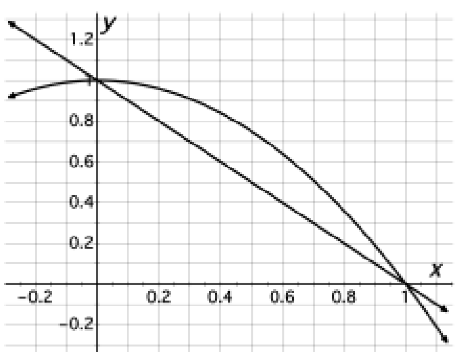
Spring 2018, Problem 10(a)

10(a) (*6 points*) Determine the area of the region bound by the graphs of $f(x) = 2x$ and $g(x) = x^{3/2}$ over the interval $[0,\ 4]$.

Fall 2018, Problem 10(a)

10. (*6 points*) Let R be the region bounded by the graphs of $y = 1 - x^2$ and $y = 1 - x$ over the interval $[0,\ 1]$. (These graphs are shown below.)

(a) Find the area of the region R.



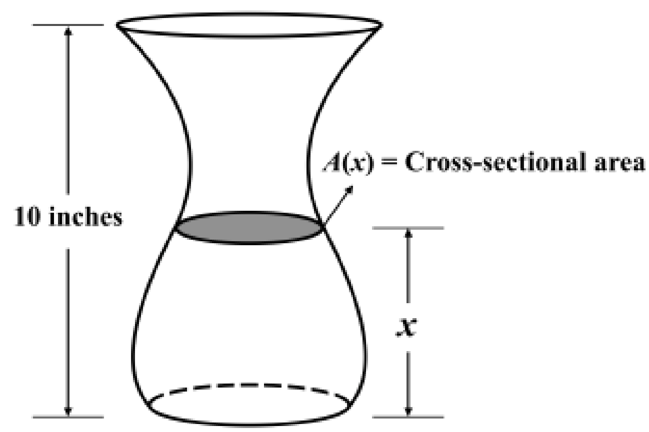
Spring 2017, Problem 11(a)

11(a) (*6 points*) Determine the area of the region between the graphs of $y = 2x + 4$ and $y = x^2 + 4$ over the interval $[0, 2]$.

Volume of a solid

Spring 2019, Problem 10(b)

10(b) (*4 points*) The vase shown below is 10 inches tall and has horizontal cross-sectional areas given by the function $A(x) = \pi\,(0.4\sin(0.7x) + 0.2x + 1)^2$, where x is the height of the respective cross section as shown in the picture. Write an integral that represents the volume of the vase. **Do not evaluate this integral.**

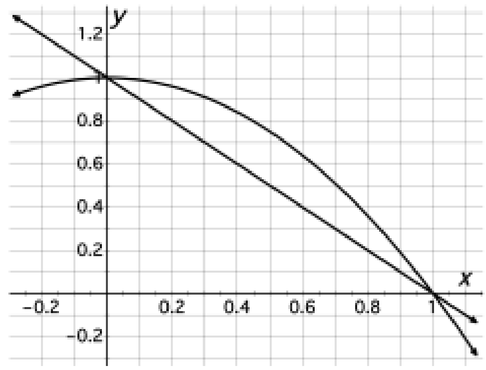


Spring 2018, Problem 10(b)

10(b) (*4 points*) Suppose the region between the graphs of $f(x) = 2x$ and $g(x) = x^{3/2}$ over the interval $[0,\ 4]$ is rotated around the x-axis. Completely set up the integral that represents the volume of the resulting solid but **do not evaluate this integral**.

Fall 2018, Problem 10(b)

10. (*6 points*) Let R be the region bounded by the graphs of $y = 1 - x^2$ and $y = 1 - x$ over the interval $[0,\ 1]$. (These graphs are shown below.)



(b) (*4 points*) Suppose the region R is rotated around the $x-$axis. Completely set up the integral that represents the volume of the resulting solid but **do not evaluate this integral**.

Spring 2017, Problem 11(b)

11(b) (*6 points*) Suppose the region between the graphs of $y = 2x + 4$ and $y = x^2 + 4$ over the interval $[0, 2]$ is rotated around the line $y = 3$. Completely set up the integral that represents the volume of the resulting solid but **do not evaluate this integral**.

Fall 2017, Problem 12(a)

12(a) (*6 points*) Determine the area of the region between the graphs of $y = 2x + 5$ and $y = x^2 + 2$ over the interval $[-1,\ 3]$.

Interpreting expressions graphically

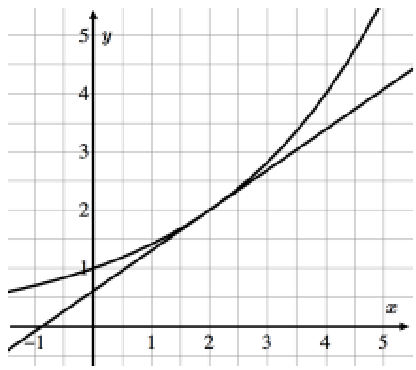
Spring 2019, Problem 11

11. (*8 points*) The graphs below are of the function g. Write an expression in each blank that represents the numerical value of the quantity identified on the corresponding graph. Select the appropriate expression for each blank from **only** the options below.

$$\frac{d}{dx}\int_2^x g(t)\,dt \qquad \sum_{k=1}^{3} g(1+k)\cdot 1 \qquad \int_1^4 g(x)\,dx \qquad \lim_{h\to 0}\frac{g(2+h)-g(2)}{h}$$

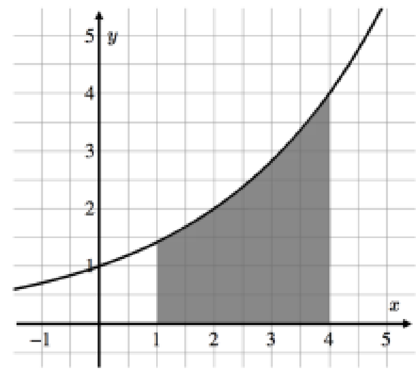
$$\int g(x)\,dx \qquad \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \qquad \sum_{k=1}^{3} g(x)\Delta x \qquad \frac{g(4)-g(0)}{4-0}$$

(a) Slope of the line tangent to the graph of g at $(2, 2)$.



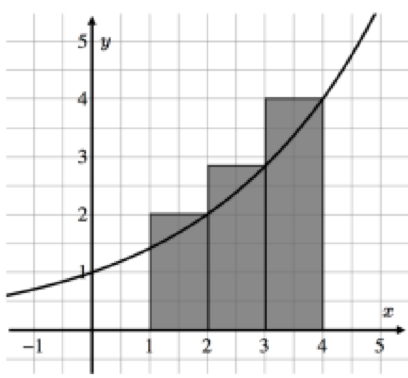
Expression: ____________________

(b) Area bound by the graph of g and the x-axis on the interval $[1, 4]$.



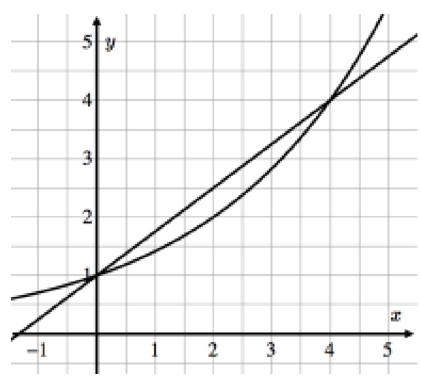
Expression: ____________________

(c) Sum of the area of the rectangles shown below.



Expression: ____________________

(d) Slope of the line secant to the graph of g which passes through the points $(0, 1)$ and $(4, 4)$.



Expression: ____________________

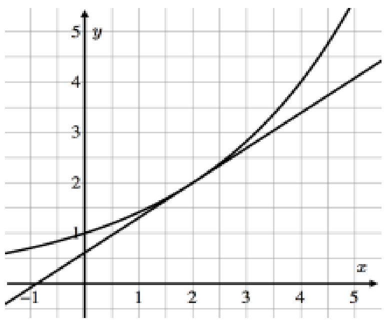
Spring 2017, Problem 12

12 (*8 points*) The graphs below are of the function g. Write an expression in each blank that represents the numerical value of the quantity identified on the corresponding graph. Select the appropriate expression for each blank from **only** the options below.

$$\frac{d}{dx}\int_2^x g(t)dt \qquad \frac{g(4)-g(0)}{4-0} \qquad \lim_{x\to 2} g(x) \qquad \int_1^4 g(x)dx \qquad R_6 \qquad \lim_{h\to 0}\frac{g(2+h)-g(2)}{h}$$

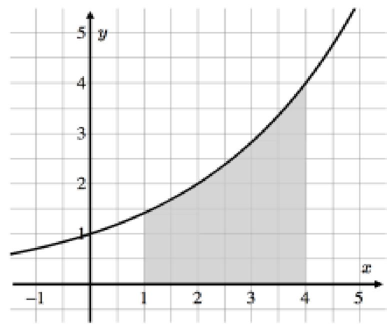
$$\int g(x)dx \qquad \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \qquad L_6 \qquad \frac{g(4)-g(1)}{4-1} \qquad \frac{dg}{dx} \qquad \sum_{i=1}^{6} g(x)$$

(a) Slope of the line tangent to the graph of g at $(2, 2)$.



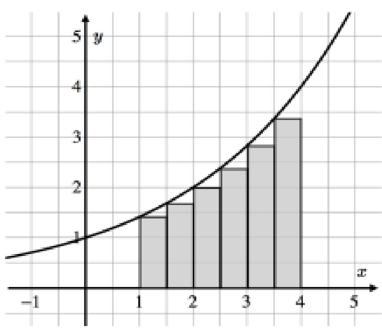
Expression: __________________________

(b) Area bound by the graph of g and the x-axis on the interval $[1, 4]$.



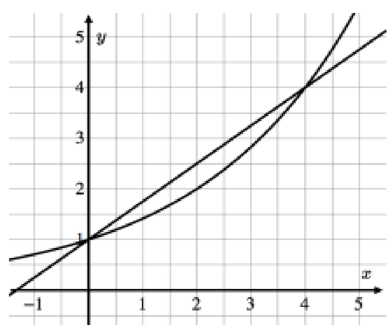
Expression: __________________________

(c) Sum of the area of the rectangles shown below.



Expression: __________________________

(d) Slope of the line secant to the graph of g which passes through the points $(0, 1)$ and $(4, 4)$.



Expression: __________________________

Fall 2017, Problem 12(b)

12(b) (*4 points*) Suppose the region between the graphs of $y = 2x + 5$ and $y = x^2 + 2$ over the interval $[-1, 3]$ is rotated around the line $y = 0$. Completely set up the integral that represents the volume of the resulting solid but **do not evaluate this integral**.