Math 2144 Exam 1 **Spring 2023** **NAME:**

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or any calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Having your phone out for any reason during the exam is an academic integrity violation. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. Reasoning which will earn credit will use material covered in the course to date.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 9 pages. Make sure all problems and pages are present.

Please turn in your notecard with the exam. Make sure your name is on your notecard.

The exam is worth 59 points in total.

You have **60 minutes** to work starting from the signal to begin.

This page is intentionally blank. You may use it for scratch work, but any work recorded on this page will not be graded.

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Suppose that a function $y = f(x)$ has a domain of all real numbers and a removable discontinuity at $x = a$. Which of the following statements must be true if the function f is continuous at all other input values?

   a. $\lim_{x\to a^-} f(x) = f(a)$ and $\lim_{x\to a^+} f(x) = f(a)$

   b. $\lim_{x\to a^-} f(x) = f(a)$ or $\lim_{x\to a^+} f(x) = f(a)$

   c. $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$

   d. $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and are not equal

   e. $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) \neq f(a)$

   (ii) Consider a continuous function defined on the interval 6.5 to 10. The function's average rate of change over that interval is 2. What is the difference between the value of the function at $x = 6.5$ and the value of the function at $x = 10$?

   a. 1.75

   b. 3.5

   c. 7

   d. 16.5

   e. 33

   (iii) Evaluate the following limit. (*Hint: Consider what the expression represents rather than algebraically manipulating the quotient.*)

$$\lim_{\Delta x\to 0} \frac{(2+\Delta x)^3 - 2^3}{\Delta x}$$

   a. 8

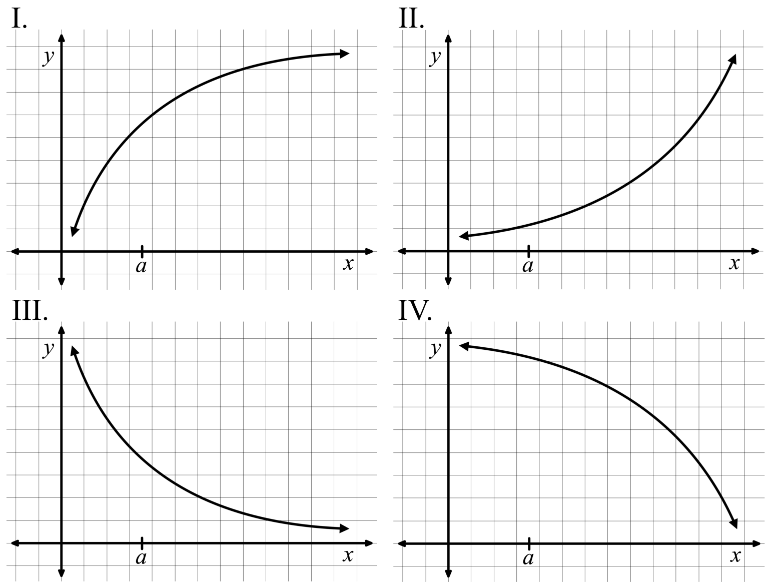
   b. 12

   c. 6

   d. 0

   e. The limit does not exist

(iv) Suppose $f'(a) > \dfrac{f(a+\Delta x) - f(a)}{\Delta x}$ for $\Delta x > 0$. Which of the following could be a graph of f?



a. I only
b. II only
c. I and IV only
d. II and III only
e. III and IV only

(v) Consider the function f defined by

$$f(x) = \begin{cases} x^4 - 6, & x < 2 \\ 10, & x = 2 \\ 4x^3, & x > 2 \end{cases}$$

Which of the following are true statements about this function?

I. f is continuous at $x = 2$

II. $\lim_{x \to 2^-} f(x) = f(2)$

III. f is differentiable at $x = 2$

IV. $\lim_{x \to 2} f(x)$ exists

a. II only
b. I and II only
c. I and IV only
d. III only
e. I, II, III, and IV

2. (*3 points each*) The Earth exerts a gravitational force of $F(r) = \frac{2.99\times10^{16}}{r^2}$ newtons (N) on an object with a mass of 75 kg located r meters from the center of the Earth. Describe the meaning of the following expressions. Your response should identify the *quantity* represented by the expression. **Specify the units** associated with each expression.

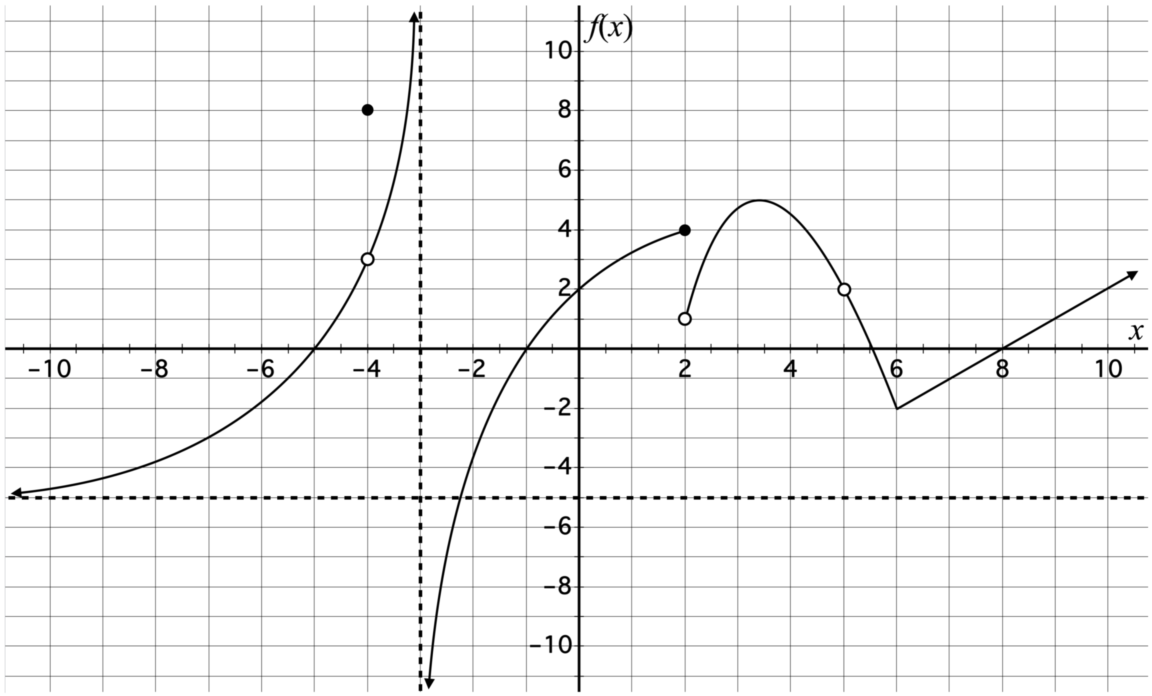
   (a) $F(6,790,000) - F(6,770,000)$.

   (b) $\dfrac{F(6,790,000) - F(6,770,000)}{6,790,000 - 6,770,000}$.

   (c) $\lim\limits_{\Delta r\to 0} \dfrac{F(6,770,000 + \Delta r) - F(6,770,000)}{\Delta r}$

   (d) What does the solution to the equation $F'(r) = -0.000193$ represent?

3(a) (*9 points*) Answer the following questions based on the graph of f below. Assume that all critical points, points of discontinuity, and end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist. Use "$\pm\infty$" as appropriate.

$\lim_{x\to 6} f(x) =$

$f'(6) =$

$\lim_{x\to 6^+} \frac{f(x)-f(6)}{x-6} =$

$\lim_{\Delta x\to 0} \frac{f(8+\Delta x)-f(8)}{\Delta x} =$

$\lim_{x\to 2^+} f(x) =$

$\lim_{x\to -3^+} f(x) =$

$\lim_{x\to -\infty} f(x) =$

$\lim_{x\to -\infty} \dfrac{df}{dx} =$

$\lim_{x\to -4} f(x) =$

3(b) (*1 point*) Identify a single value of x in the interval $[-10, 10]$ at which f is continuous but not differentiable.

$x =$ ________

4. (*6 points*) Use the limit definition of derivative to show that

$$\frac{d}{dx}\left(x - x^2\right) = 1 - 2x.$$

You **must** use the **limit definition of derivative** to receive credit. You will not earn credit for applying rules of differentiation.

5. (*5 points*) Without referring to a graph, determine whether the function

$$f(x) = \ln(x) - \cos(x)$$

has at least one real zero (or x-intercept) in the interval $(1, 3)$. Justify your response by either explaining your reasoning or citing a theorem from the text. If you cite a theorem, you need to explicitly state that the hypotheses of the theorem are satisfied.

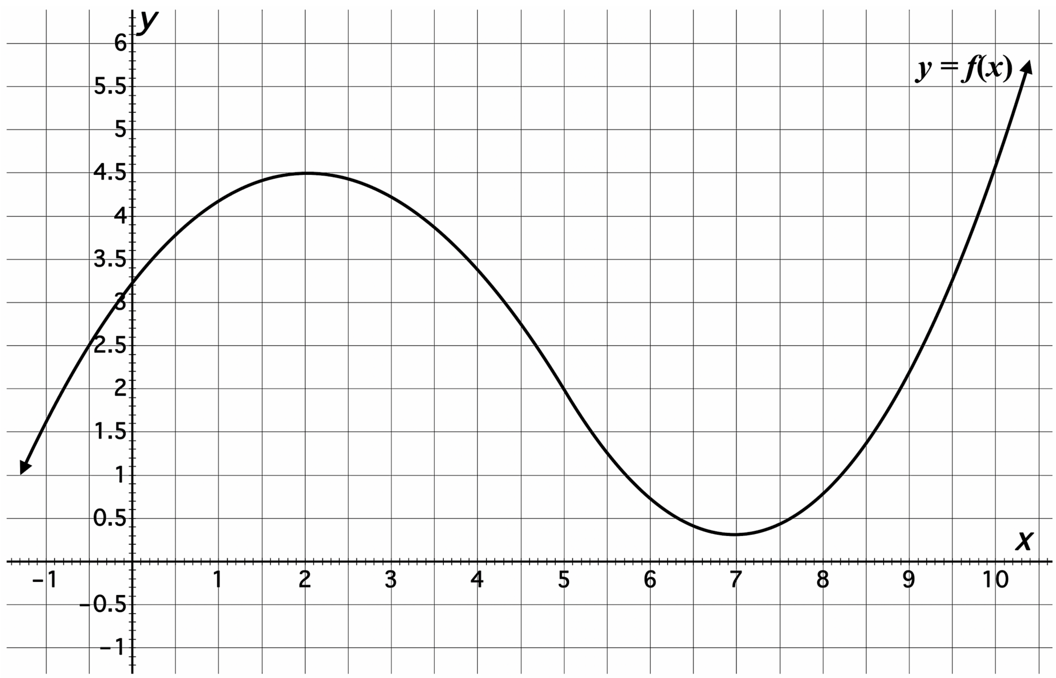
6. (*4 points each*) Compute the following derivatives. **You do not need to simplify** after taking each derivative.

   (a) Let $f(x) = 5x^6 - 4x^3 - 9x^2 - 16$. Find $f'(x)$.

   (b) Let $y = \dfrac{\sqrt{x}}{x^2+1}$. Find $\dfrac{dy}{dx}$.

   (c) Let $g(x) = x^{-2}e^x$. Find $g'(x)$.

7. (*2 points each*) The graph of the function $y = f(x)$ is given below. Determine values for a, b, and c that satisfy the respective inequality or equality. (*Note that there are multiple values for b, and c that satisfy these (in)equalities.*)



(a) Determine a single numerical value for the constant b that makes the following inequality true:

$$0 < \frac{f(b+1) - f(b)}{1} < 1$$

$b =$ ________

(b) Determine a single numerical value for the constant c that makes the following equality true:

$$\lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = 0$$

$c =$ ________