# Calculus II Final Exam Practice Problems

1. (a) Sketch the conic section. Find and label any foci, vertices, and asymptotes.
$(x-3)^2 - 9y^2 = 36$
(b) Find the equation of the ellipse with foci $(0,\pm 2)$ and semi-major axis length 3.

2. (a) Find the area of one petal of the rose $r = 4\sin(3\theta)$.
(b) Find the arc length of the Archimedean spiral $r = 2\theta$, from $\theta = 0$ to $\theta = \pi/2$.

3. Use the formula $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ for $|x| < 1$ to find the number of terms N such that the difference between the Nth partial sum $S_N$ estimating ln(1.1) is less than $10^{-2}$.

4. Find the Taylor series about $x_o=\pi/4$ for $f(x) = \sin(x)$ .

5. Find the radius of convergence and interval of convergence for

(a) $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k!}$ (b) $\sum_{k=1}^{\infty} 3 \cdot 2^k \cdot x^k$

6. Classify each series as absolutely convergent, conditionally convergent, or divergent.
Show your work. (You can use any method – you do not have to use the hints.)

(a) $\sum_{k=1}^{\infty} \left(\frac{\ln(k)}{k+2}\right)^k$ (hint: root test)

(b) $\sum_{k=1}^{\infty} \frac{3}{2k^{1.5}-1}$ (hint: comparison or limit comparison test)

(c) $\sum_{k=1}^{\infty} \frac{3^{2k}}{3k!}$ (hint: ratio test)

(d) $\sum_{k=1}^{\infty} \frac{2k}{(k^2+1)^3}$ (hint: integral test with substitution $u=k^2 + 1$)

(e) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{(k+1)}$ (hint: divergence test)

(f) $\sum_{k=1}^{\infty} \frac{2}{\sqrt[5]{k^3+1}}$ (hint: limit comparison with a p-series)

7. Write out the first five terms of the sequence, determine whether the sequence converges, and if so find its limit.

(a) $\left\{\frac{2n+3}{7n+1}\right\}_{n=1}^{\infty}$

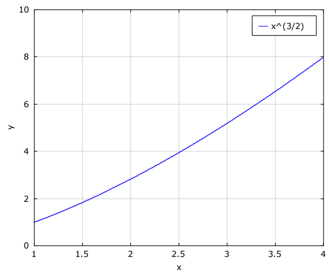
(b) $\left\{\frac{n+3}{n^2+1}\right\}_{n=1}^{\infty}$

8. Find the sum of each series.

(a) $\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$ (Hint: Telescoping)

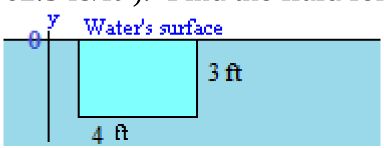
(b) $\sum_{k=1}^{\infty} \frac{5}{3 \cdot 2^k}$ (Hint: Geometric)

9. Find the arc length of the curve $y = x^{3/2}$ from x=1 to x=4.




10. Find the exact area of the surface generated when $y = 2x^{1/2}$ from x = 1 to x = 4 is revolved about the x-axis.

11. The lamina below is submerged vertically in water (of weight density 62.5 lb/ft$^3$). Find the fluid force against it.




12. Evaluate the integrals. Show your work.

(a) $\int \frac{x^2}{\sqrt{4-x^2}} dx$ (Hint: Trig. substitution.)

(b) $\int_1^{\infty} \frac{1}{1+t^2} dt$ (Hint: Derivative of $\tan^{-1}t$)

(c) $\int_{\pi/4}^{\pi/2} \tan^3(2x)\sec^3(2x)dx$

(d) $\int_0^{\pi} \sqrt{1+\cos(x)}\, dx$ (Hint: Half-angle identity.)

(e) $\int \frac{\ln(x)}{x} dx$ (Hint: Integration by parts.)

(f) $\int x^2 e^x dx$ (Hint: Two integrations by parts.)

(g) $\int x \sinh(x^2) dx$ (Hint: Substitution.)

(h) $\int \frac{2x^2}{x^2-4x+3} dx$ (Hint: Polynomial division and partial fractions.)

13. Solve the differential equation and apply the stated initial value.

$$(y-1)^2 \frac{dy}{dt} = -2t \quad , \quad y(0) = 2$$

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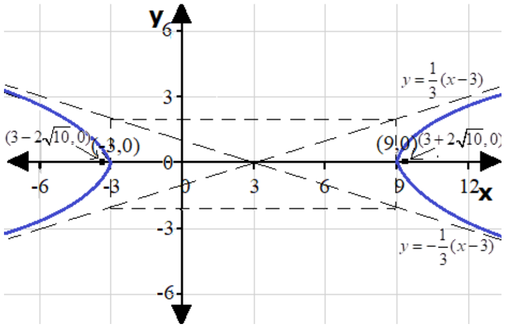
# Answers

1. a.




Vertices: (-3,0), (9,0) , Foci: $(3-2\sqrt{10},0)$ , $(3+2\sqrt{10},0)$ , Asymptotes: $y=\pm\frac{1}{3}(x-3)$

b. $\left(\frac{y}{3}\right)^2+\left(\frac{x}{\sqrt{5}}\right)^2=1$

2. a. 4π/3 b. $\frac{\pi\sqrt{\pi^2+4}}{4}+\ln\left(\frac{\pi}{2}+\sqrt{\frac{\pi^2}{4}+1}\right)$

3. 1 (The alternating series error theorem says that the error is no more than the absolute value of the next term: $|-(.1)^2/2| = .005$.)

4. $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\frac{\sqrt{2}}{2}}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{\frac{\sqrt{2}}{2}}{3!}\left(x-\frac{\pi}{4}\right)^3+\frac{\frac{\sqrt{2}}{2}}{4!}\left(x-\frac{\pi}{4}\right)^4+\frac{\frac{\sqrt{2}}{2}}{5!}\left(x-\frac{\pi}{4}\right)^5-\ldots$

5. a. R = ∞ , (–∞,∞) b. R = ½ , (– ½ , ½ )
6. a. Converges absolutely b. Converges absolutely c. Converges absolutely
   d. Converges absolutely e. Diverges f. Diverges
7. a. 2/7 b. 0
8. a. 1/3 b. 5/3
9. $\frac{8}{27}\left(10^{3/2}-\frac{13^{3/2}}{8}\right)$
10. $\int_1^4 2\pi\cdot 2\sqrt{x}\sqrt{1+(1/\sqrt{x})^2}\,dx=4\pi\int_1^4\sqrt{x+1}\,dx=4\pi\int_2^5\sqrt{u}\,du=\frac{8\pi(5^{3/2}-2^{3/2})}{3}$
11. 1,125 lb.
12. a. $2\sin^{-1}(x/2)-\sin(2\sin^{-1}(x/2))$ + C b. π/4 c. Diverges
    d. $2\sqrt{2}$ e. $(\ln(x))^2/2$ + C f. $x^2e^x - 2xe^x + 2e^x + C$
    g. $\frac{1}{2}\cosh(x^2)+C$ h. $4\ln|x^2 - 4x + 3| - 5\ln|x - 1| + 5\ln|x - 3| + 2x + C$
13. $y=1+\sqrt[3]{1-3t^2}$