The Spring 2025 Differentiation Assessment will cover the following sections:

- 3.3 Product and Quotient Rules
- 3.6 Derivatives of Trig Functions
- 3.7 Chain Rule
- 3.8 Implicit Differentiation
- 3.9 Derivatives of General Exponential and Logarithmic Functions
- 3.10 Related Rates

Here is a sampling of problems from past exams. You should also look for practice problems from your class notes, textbook, and online homework to help you study.

From SP23 Exam 2

4. (*4 points each*) Compute the following derivatives. Do not simplify.

(a) Let $f(\theta) = \dfrac{e^{\sin(\theta)}}{\theta}$. Find $f'(\theta)$.

(b) Let $y = 2^{x^2}$. Find the **second derivative** $\left(\frac{d^2y}{dx^2}\right)$ of y with respect to x.

5. (*6 points*) Find $\frac{dy}{dx}$ for the curve $e^{3x} = \sin(y) - 8y$. Solve for $\frac{dy}{dx}$ in terms of both x and y.

6. (*20 points*) One boat approaches a dock from the west at a constant speed of 20 km/hr. At 12:00 PM, this boat is 60 km directly west of the dock. Another boat leaves the same dock at 11:00 AM traveling directly north at a constant speed of 15 km/hr.

   (a) (*10 points*) At what rate (in units of km/hr) is the distance between the boats changing at 1:00 PM? Work must be shown to receive credit.

From SP22 Ex 2

4. (*4 points each*) Compute the following derivatives. Do not simplify.

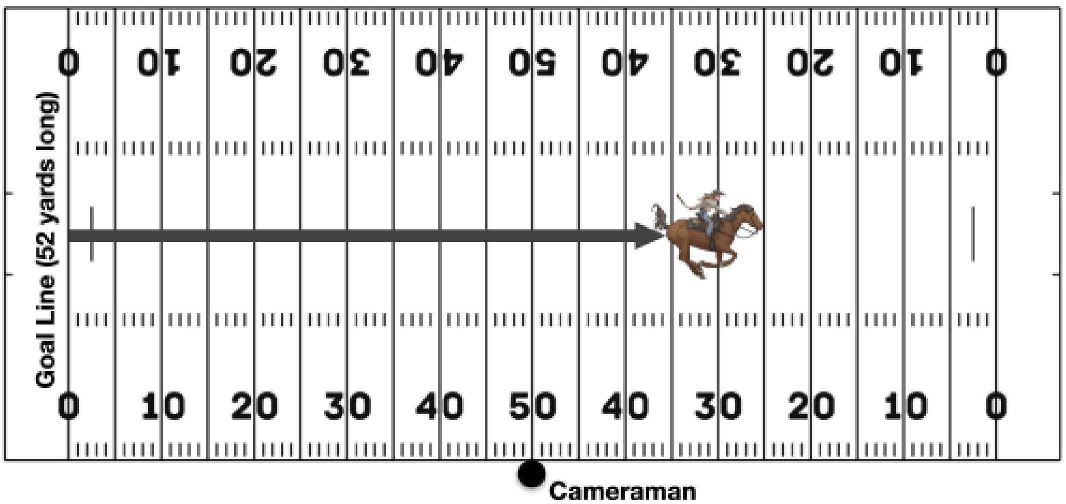
   (a) Let $f(x) = \arctan(x^3)$. Find $f'(x)$.

   (b) Let $y = 2^{\sin(\theta)}$. Find $\frac{dy}{d\theta}$.

   (c) Let $g(t) = \log_7(t^2)$. Find $g''(t)$.

5. (*6 points*) Find $\frac{dy}{dx}$ for the curve $\sqrt{x} - \cos(y) = e^y + \pi$. Your answer for $\frac{dy}{dx}$ should contain both x and y.

6. (*10 points*) After the OSU football team scores a touchdown during a home game, a horse named Bullet runs down the center of the field at a constant speed of 18 yards per second. A cameraman standing at the 50 yard-line films bullet running (see image below). At what rate must the cameraman rotate his camera to keep Bullet in the frame at exactly 4 seconds after bullet passed the goal line (see image below)? Provide an answer accurate to three decimal places.



From FA22 Ex2

4. (*4 points each*) Compute the following derivatives. Do not simplify.

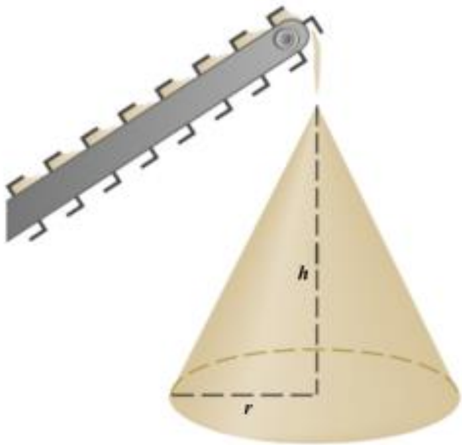
   (a) Let $f(t) = \sec(\sqrt{2t})$. Find $f'(t)$.

   (b) Let $y = 5^{\arcsin(x)}$. Find $\frac{dy}{dx}$. (*Recall that* $\arcsin(x) = \sin^{-1}(x)$, *the inverse of the sine function).*

   (c) Let $g(t) = x\ln(x)$. Find the **second derivative** of g $\left(\frac{d^2g}{dt} \text{ or } g''(x)\right)$.

5. (*6 points*) Find $\frac{dy}{dx}$ for the curve $xe^y = \tan(y)$. Your answer for $\frac{dy}{dx}$ should contain both x and y.

6. (*10 points*) Suppose gravel is being poured into a conical pile at a rate of 5 m^3/s, and suppose that the radius r of this cone is always half its height h. How fast is the height of the pile increasing when the height is 10 m?
(*Note that the formula for the volume of a cone is* $V = \frac{1}{3}\pi r^2 h$).



From FA23 Ex 2

(iv) A lighthouse is located on a small island 3 kilometers away from the nearest point P on a straight shoreline. Let x represent the distance between P and the light beam's intersection with the shoreline. Also let θ represent the measure of the angle created by the beam of light and the line connecting the lighthouse and P. Which formula defines the relationship between the rate of change of the angle's measure and the rate at which the beam of light is moving along the shoreline?

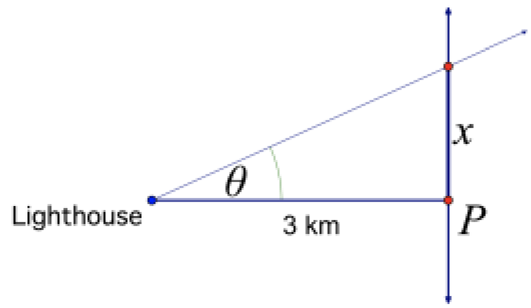
a. $\tan\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$

b. $\sec^2(\theta) \cdot \frac{d\theta}{dt} = \frac{1}{3} \cdot \frac{dx}{dt}$

c. $\sec^2\left(\frac{d\theta}{dt}\right) = \frac{1}{3} \cdot \frac{dx}{dt}$

d. $\frac{d\theta}{dt} = \frac{1}{3\sec^2(\theta)}$

e. $\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{dx}{dt}\right)^2}$



4. (*4 points each*) Compute the following derivatives.

(a) Let $f(x) = \frac{7^x}{x^7}$. Find $f'(x)$.

(b) Let $y = \ln(\sec(x))$. Find the **second derivative** $\left(\frac{d^2y}{dx^2}\right)$, or y'', of y with respect to x.

5. (*6 points*) Find $\frac{dy}{dx}$ for the curve $\ln\left(3x^4\right) = e^{\sin(y)} + y$. Solve for $\frac{dy}{dx}$ in terms of both x and y.

6. (*10 points*) Firefighter Everett raises the ladder of a firetruck to the third floor of a burning building. As the ladder elevates, the measure of the angle, θ, the ladder makes with the horizontal changes at a constant rate of $\frac{\pi}{20}$ radians per second. At what rate does the slope of the ladder change when $\theta = \frac{\pi}{6}$?

Express your solution as an exact value, not an approximation.

(*Hint: Consider which of* $\sin(\theta)$, $\cos(\theta)$, *or* $\tan(\theta)$ *represents the slope of the ladder.*)

From FA24 Ex 2

3. (*5 points each*) Compute the following derivatives. Do not simplify.

(a) Let $g(x) = 6x^4 + \ln\left(\sqrt{x} - 4\right) + 8x$. Find $g'(x)$.

(b) Let $y = \dfrac{e^x}{\sin(x)}$. Find $\dfrac{dy}{dx}$.

(c) Find $\dfrac{dy}{dx}$ when $y = \arctan(x^3)$ in terms of x only.

4. (*8 points*) Find $\frac{dy}{dx}$ for the curve $x^2 - \sin(y) = xy^2 + e^\pi$. You must solve for $\frac{dy}{dx}$ in terms of x and y.

6. (*10 points*) An airplane is flying horizontally, parallel to the ground, in a straight line at an altitude of 8 kilometers and passes directly over a radar antenna. When the distance between the plane and the antenna is 12 kilometers, the radar detects that the distance between the plane and the antenna is changing at a rate of 340 kilometers per hour. What is the speed of the airplane at that moment? (Express your solution as an exact answer.)

From SP24 Ex 2

4. (*5 points each*) Use the table of values below to answer the following questions. State your answers as exact values.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	7	-1	3	6
2	5	1	3	-4
4	0	1	0	-2

(a) If $h(x) = f((g(x))^2)$, then $h'(2) =$

(b) If $s(x) = x^2 \arctan(f(x))$, then $s'(4) =$

(c) If $k(x) = 2^{g(x)}$, then $k'(1) =$

5. (*4 points each*) Compute the following derivatives.

   (a) Let $f(x) = \pi x^{e^{\pi}}$. Find $f'(x)$.

   (b) Let $y = \sin(\theta)$ where θ is measured in degrees (not radians!). Find the **second derivative** $\left(\frac{d^2y}{d\theta^2}\right)$, or y'', of y with respect to θ.

6. (*6 points*) Find $\dfrac{dy}{dx}$ for the curve $y - \sec(4^y) = xy + \sin(x)$. Solve for $\dfrac{dy}{dx}$ in terms of both x and y.