

Oklahoma State University
Fall 2022

Math 2153– Final Exam- preparation
Comprehensive

Note: This preparation is not a complete pool.

Instructor: Dr. Cuiyu He

09/22/2022

Name: _____
OSU ID Number: _____

This exam contains 29 pages (including this cover page) and 20 questions. Total of points is 0.
Good luck and Happy reading work!

Distribution of Marks

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
9	0	
10	0	
11	0	
12	0	
13	0	
14	0	
15	0	
16	0	
17	0	
18	0	
19	0	
20	0	
Total:	0	

1 integration techniques

1. Evaluate the indefinite integrals using substitution.

(a)

$$\int \frac{e^x}{(e^x - 1)^6} dx$$

(b)

$$\int \frac{1}{x \ln^4(3x)} dx$$

(c)

$$\int \sin(x + 2) \cos(x + 2)^5 dx$$

(d)

$$\int \sec(u)^2 \tan(u)^4 du$$

(e)

$$\int \sec(x)^8 \tan(x)^3 dx$$

(f)

$$\int \sec(u)^2 \tan(u)^4 du$$

2. Evaluate the indefinite or definite integrals using integration by parts.

(a)

$$\int \sin^2(x) dx$$

(b)

$$\int (3x + 2)e^x dx$$

(c)

$$\int x^2 \cos(13x) dx$$

(d)

$$\int \frac{\ln(x)}{x^9} dx$$

(e)

$$\int e^x \cos(6x) dx$$

3. Evaluate the indefinite or definite integrals using your choice's proper method(s).

(a)

$$\int \frac{dx}{\sqrt{49 - x^2}}$$

(b)

$$\int \frac{x^2}{\sqrt{225 - x^2}} dx$$

(c)

$$\int x^3 \sqrt{x^2 - 4} dx$$

(d)

$$\int \frac{dx}{(9x^2 + 64)^2}$$

(e)

$$\int \frac{1}{(x-3)(x-5)} dx$$

(f)

$$\int \frac{6x - 2}{x^2 - 11x + 30} dx$$

(g)

$$\int \frac{x^2 - 10x + 1}{x^2 + x} dx$$

(h)

$$\int \frac{20x^2 + 41x + 11}{(x-1)(x+1)^2} dx$$

(i)

$$\int \frac{5}{x^2(x^2 + 25)} dx$$

4. (a) Find M_6 for $\int_{-3}^1 xe^x dx$ using midpoint rule.

(b) Evaluate the actual value for $\int_{-3}^1 xe^x dx$ and compute the error for M_6 .

(c) At least how many points, N , are needed so to make $error(M_N) \leq 0.001$?

Hint:

$$Error(M_N) = \left| \int_a^b f(x) dx - M_N \right| \leq K_2 \frac{(b-a)^3}{24N^2}$$

(d) Find T_6 for $\int_{-3}^1 xe^x dx$ using Trapezoid rule.

(e) Find S_6 for $\int_{-3}^1 xe^x dx$ using Simpson's rule.

5. A plate in the shape of an isosceles triangle with a base of 1 m and height 7 m is submerged vertically in a tank of water so that its vertex touches the surface of the water. Calculate the total fluid force on one side of the plate. ($\rho = 1000\text{kg}/\text{m}^3, g = 9.8\text{m}/\text{s}^2$).

2 Sequences, series, and power series

6. (a) Calculate the first four terms of the sequence $\{a_n\}$, starting with $n = 1$.

$$a_n = \frac{1}{n^2 + 1}.$$

$$a_1 =$$

$$a_2 =$$

$$a_3 =$$

$$a_4 =$$

- (b) Use the Theorem of Sequence Defined by a Function to determine the limit of the sequence or state that the sequence diverges, i.e., find

$$\lim_{n \rightarrow \infty} a_n =$$

7. (a) Use the appropriate limit laws and theorems to determine the limit of the sequence.

$$a_n = \frac{n}{4} \sin(4/n)$$

(b) Determine whether the following series converges or not using the divergence test.

$$\sum_{n=1}^{\infty} a_n$$

8. Given the following series

$$\sum_{n=1}^{\infty} \frac{1}{n} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

(a) Find the first four terms of the partial sum sequence for each series.

(b) Use Integration Test to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

(c) Use Test for Alternating Series to show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges.

9. Use the proper test to determine whether the following series converges or diverges.

(a)

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

(b)

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - 1}$$

(c)

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

(d)

$$\sum_{n=1}^{\infty} \frac{1}{2^n + n}$$

(e)

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

10. Determine whether the following series converges conditionally, absolutely or diverges.

(a)

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln(n)}.$$

(b)

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}.$$

11. Find the value of the series S if it converges.

$$S = \sum_{n=2}^{\infty} \frac{1}{3^n}.$$

12. Given the following power series.

$$\sum_{n=1}^{\infty} \frac{3^n}{(n^3 + 2n + 10)5^n} x^n$$

(a) Find the radius of convergence of the power series using the Ratio Test.

(b) Find the interval of convergence for the power series.

13. Given the following power series.

$$\sum_{n=1}^{\infty} \frac{100^n}{n!} x^n.$$

(a) Find the radius of convergence of the power series using the Ratio Test.

(b) Find the interval of convergence for the power series.

14. (a) Find the first four terms of the Maclaurin series for $\frac{1}{1-x}$ **based on its definition**. Hint:

The formula of Maclaurin polynomial read $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

- (b) Based on (a), find the general formula for $\frac{f^{(n)}(0)}{n!}$, $n \geq 0$.

- (c) Based on (b), write the Maclaurin series for $\frac{1}{1-x}$ and state its interval of convergence.

15. (a) Write the Maclaurin series for $\sin(x)$ and state its interval of convergence.

(b) Find the Maclaurin series for $\sin(x^2)$ and state its interval of convergence.

(c) Find the Maclaurin series for $\cos(x)$ based on the fact that $\cos(x) = \frac{d}{dx} \sin(x)$.

16. (a) Find the power series for $\ln(x + 1)$.

Hint: First find the power series for $\frac{1}{x + 1}$, then integrate.

(b) Find the power series for $\frac{8x}{(x^2 + 1)^2}$.

Hint: First find the power series for $\frac{-4}{1 + x^2}$, then differentiate.

3 Parametric equations and its applications

17. A particle moves following the parametric equations $x(t) = 2 + 3t$, $y(t) = 5t$, $t \in [0, 2]$. Assume (x, y) in meters and t in seconds.
- (a) Plot the trajectory of the particle.

(b) Find the tangent line of the trajectory at $t = 1$.

(c) Find the speed of the particle at $t = 1$.

(d) Find the traveled distance of the particle in the first 2 seconds.

18. A particle follows the trajectory $x(t) = \frac{2\sqrt{8}}{3}t^{3/2}$, $y(t) = 2t - t^2$ with t in seconds and distance in centimeters. Answer the following questions:

(a) What is the initial position of the particle?

(b) What is the initial velocity of the particle?

(c) What is the initial speed of the particle?

(d) When does the particle reach its maximum height?

(e) What is the particle's maximal height?

(f) What is the particle's speed when it reaches its maximum height?

(g) When does the particle reach the ground?

(h) What is the particle's speed when it reaches the ground?

(i) What is the total distance travelled by the particle?

19. Find a parametric equations for the following equations.

(a)

$$x^2 + y^2 = 1.$$

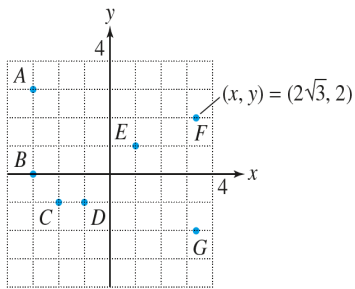
(b)

$$\frac{(x - 2)^2}{9} + \frac{y^2}{16} = 1$$

(c)

$$y = 2x + 1.$$

20. (a) Match the points plotted on the graph with their polar coordinates.



- (b) Convert from polar to rectangular coordinates and plot the point on the above coordinate system.

$$\left(3, \frac{7\pi}{6}\right), \quad \left(-3, \frac{7\pi}{6}\right)$$

- (c) Which of the following are possible polar coordinates for the point P with rectangular coordinates $(0, -12)$.

- $\left(-12, -\frac{3\pi}{2}\right)$
- $\left(12, \frac{\pi}{2}\right)$
- $\left(12, -\frac{3\pi}{2}\right)$
- $\left(12, -\frac{7\pi}{2}\right)$
- $\left(-12, -\frac{\pi}{2}\right)$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.