From Spring 2025 Final Exam

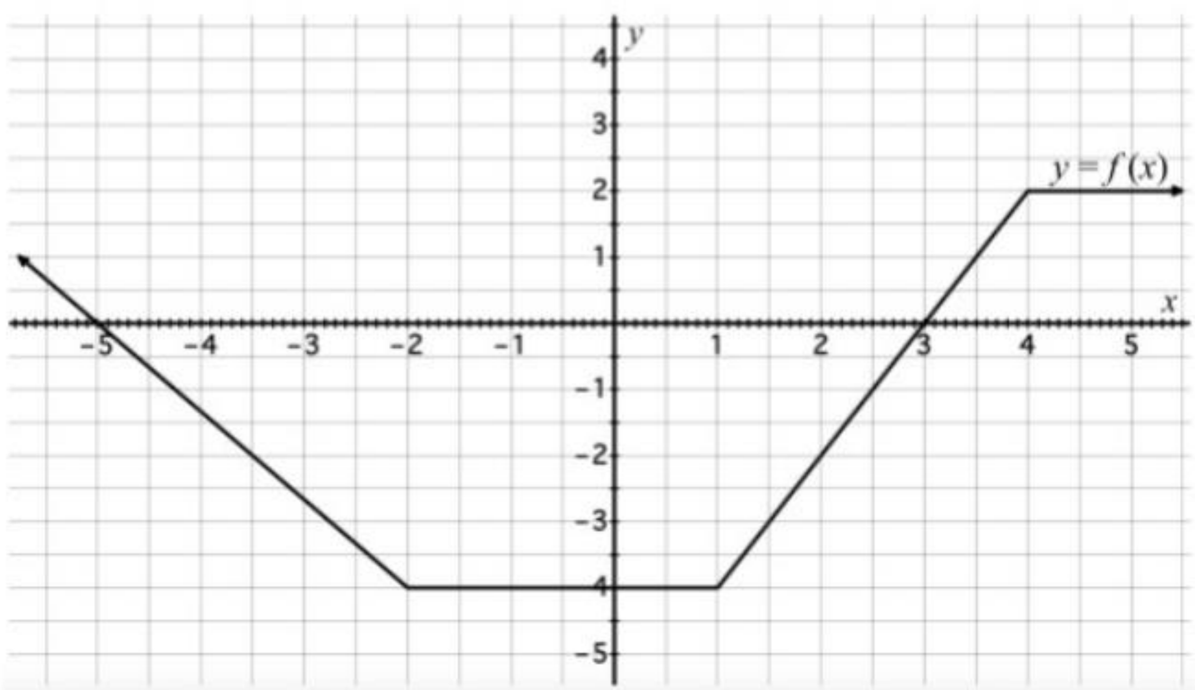
(vi) The rate of rainfall accumulation, $A(t)$, in inches per hour, is a function of the time t in hours since the rain began. Data from a recent storm are shown in the table.

t	1	2	4	7
$A(t)$	0.5	0.6	0.4	0.1

Compute a right-hand Riemann sum from the given data, and interpret the result in context.

a. $R_3 = 1.7$ is an approximation of the total amount of rainfall in inches.
b. $R_3 = 2.9$ is an approximation of the total amount of rainfall in inches.
c. $R_3 = 1.1$ is an approximation of the rate of rainfall in inches per hour.
d. $R_3 = 1.7$ is an approximation of the rate of rainfall in inches per hour.
e. $R_3 = 1.1$ is an approximation of the total amount of rainfall in inches.

The graph below will be used for parts (vii) and (viii)



(vii) Consider the area function $A(x) = \int_{-1}^{x} f(t)dt$. Using the given graph of $f(t)$, compute the value of $A(4)$.

a. -12
b. 0
c. 12
d. -7
e. -11

(viii) Consider the area function $A(x) = \int_{-1}^{x} f(t)dt$. Using the given graph of $f(t)$, compute the value of $A'(1)$.

a. -4
b. 2
c. 0
d. -2
e. DNE

7. (*28 points*) Compute the following integrals. Give exact answers on parts (iii) and (iv).

(a) $\displaystyle\int (4x^5 - 3x)dx$

(b) $\displaystyle\int (\cos(x) + \pi)dx$

(c) $\displaystyle\int_0^1 e^x dx$

(d) $\displaystyle\int_2^5 \frac{1}{x} dx$

8. (*28 points*) Evaluate each of the following. Show all your work and give exact answers. If the answer is a number, do not round. If the integral is not a basic integral, then you MUST show your work to get full credit.

(a) $\displaystyle\int y^5 \sqrt{y^6 - 4} dy$

(b) $\displaystyle\int \frac{(\ln(x))^9}{x} dx$

(c) $\dfrac{d}{dx}\displaystyle\int_0^{x^3} \cot(\ln(t))dt$

(d) $\displaystyle\int_1^4 12x(x^2-1)^5 dx$

From Fall 2024 Final Exam

(iv) If $\displaystyle\int_a^b g(x)\,dx = 4a+b$, then $\displaystyle\int_a^b (g(x)+7)\;dx =$

a. $8b-11a$

b. $8b+11a$

c. $8b-3a$

d. $7b-7a$

e. $4a+b+7$

5. (*5 points each*) Evaluate each of the following. Show all your work and give **exact** answers. If the answer is a number, **do not round**. If the integral is not a basic integral, then you MUST show your work to get full credit.

   (a) $\displaystyle\int \left(10x^7 - \frac{16}{x}\right) dx$

   (b) $\displaystyle\int t^2 \cdot \cos(t^3)\, dt$

   (c) $\displaystyle\frac{d}{dx}\int_{\frac{\pi}{2}}^{2x} \sin(\theta^2)\, d\theta$

   (d) $\displaystyle\int_1^4 \frac{5\sqrt{x}}{\sqrt{x}}\, dx$

6. (*5 points each*) The velocity (in meters per second) of a car t seconds after passing a crosswalk is given by the function $v(t) = \frac{3}{4}t^2 - 4t + 6$.

(b) Compute the car's distance from the crosswalk five seconds after the car passed the crosswalk. (*Note that when* $t = 0$*, the car is 0 meters past the crosswalk.*)

8. (*3 points each*) The population $P(t)$ of a small town changes at a rate $R(t) = \dfrac{dP}{dt}$ given by the function below, where t is measured in months since January 1, 2024.

$$R(t) = \frac{dP}{dt} = \frac{4950e^{-0.05t}}{(1+99e^{-0.05t})^2}$$

(a) Write, but **do not evaluate**, L_{10} using summation notation for the function $y = R(t)$ over the interval $[4, 9]$.

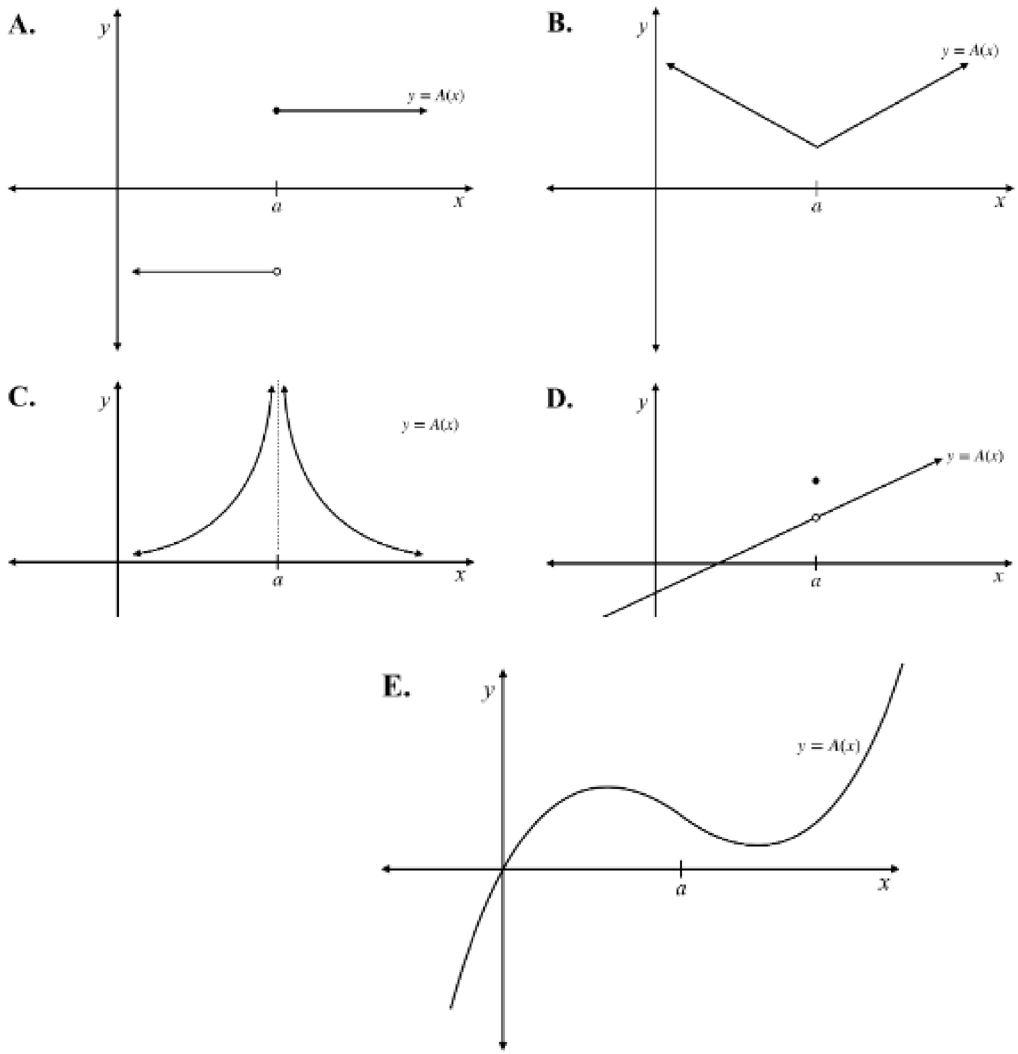
(b) Write, but **do not evaluate**, the definite integral whose value is approximated by the following Riemann sum:

$$\frac{1}{20} \cdot \sum_{k=0}^{39} R\left(5 + \frac{k}{20}\right)$$

(c) Explain what the definite integral you wrote in Part (b) represents in the context of the town's population.

From the Spring 2024 Final Exam

(ii) Let $A(x) = \int_{-1}^{x} f(t)\,dt$. Suppose $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist but are not equal. If $\lim_{x\to a} A(x)$ exists, which of the following could be the graph of $y = A(x)$?



A. Graph A

B. Graph B

C. Graph C

D. Graph D

E. Graph E

(iii) Oil leaks out of a tank at a rate of $r = f(t)$ gallons per minute, where t is measured in minutes. Which of the following expressions represents the **exact** amount of oil that leaked out of the tank from 15 to 45 minutes after oil started leaking?

I. $f(45) - f(15)$

II. $\displaystyle\int_{15}^{45} f(t)\,dt$

III. $\displaystyle\sum_{k=1}^{60} f\left(15 + \frac{k}{2}\right) \cdot \frac{1}{2}$

IV. $\displaystyle\lim_{N\to\infty} \sum_{k=1}^{N} f\left(15 + \frac{30k}{N}\right) \cdot \frac{30}{N}$

a. I only

b. I and II only

c. II and IV only

d. I, II, and IV only

e. I, II, III, and IV

(iv) $\displaystyle\frac{d}{dx}\int_{2}^{h(x)} f(t)\,dt =$

a. $f\left(h(x)\right) \cdot h'(x)$

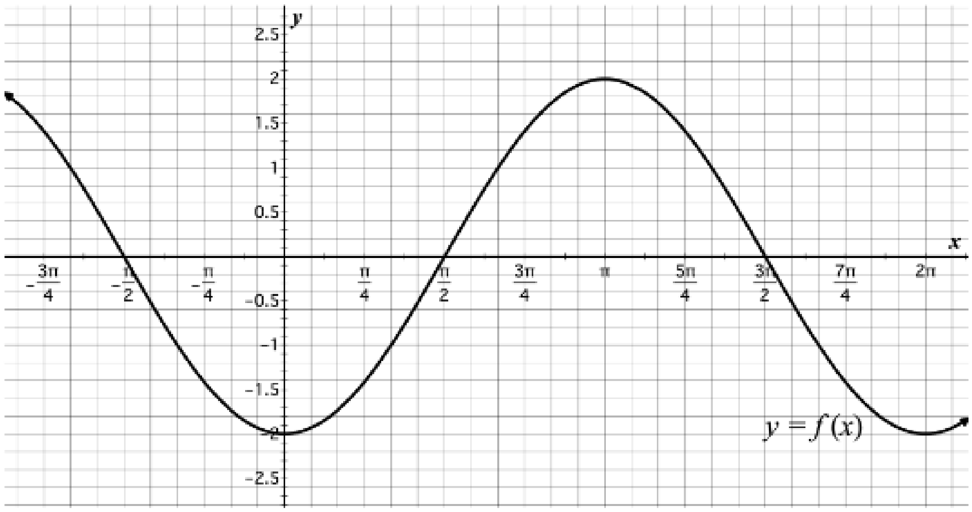
b. $f'(h(x)) \cdot h'(x)$

c. $f\left(h(x)\right) - f(2)$

d. $f\left(h(x)\right)$

e. $f'(h(x))$

(v) The graph of the function f is below. The following inequalities compare the values of the quantities $f'\left(\frac{\pi}{2}\right)$, $f''(\pi)$, $\dfrac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$, and $\displaystyle\int_{\pi}^{2\pi} f(x)\,dx$. Which of the following string of inequalities is true?



a. $\displaystyle\int_{\pi}^{2\pi} f(x)\,dx > f''(\pi) > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right)$

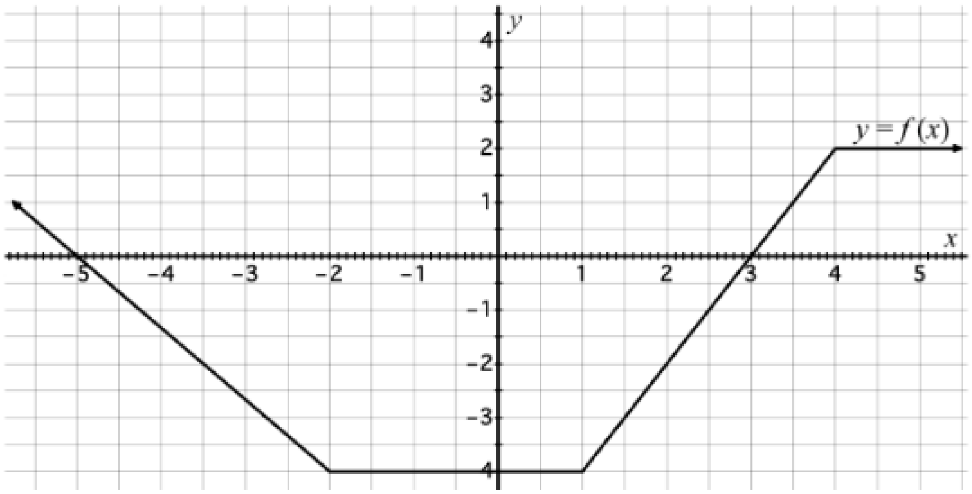
b. $\displaystyle f'\left(\frac{\pi}{2}\right) > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > \int_{\pi}^{2\pi} f(x)\,dx > f''(\pi)$

c. $\displaystyle \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right) > \int_{\pi}^{2\pi} f(x)\,dx > f''(\pi)$

d. $\displaystyle f''(\pi) > \int_{\pi}^{2\pi} f(x)\,dx > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}} > f'\left(\frac{\pi}{2}\right)$

e. $\displaystyle f'\left(\frac{\pi}{2}\right) > f''(\pi) > \int_{\pi}^{2\pi} f(x)\,dx > \frac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$

(vi) Let the function $A(x) = \int_0^x f(t)\,dt$, where f is the function graphed below. Which of the following are the correct values for $A(-2)$ and $A''(3)$?



a. $A(-2) = -8$ and $A''(3) = 0$

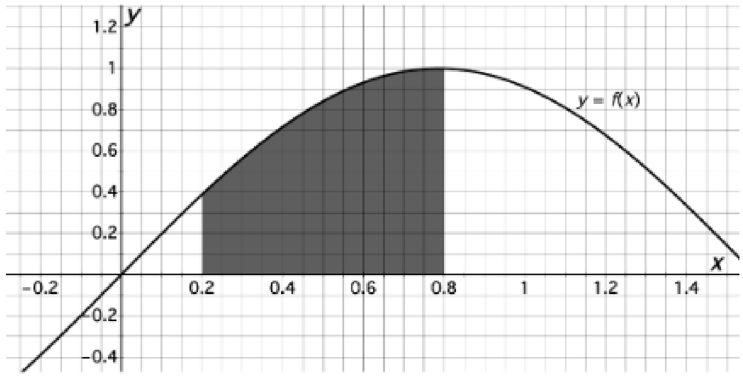
b. $A(-2) = 8$ and $A''(3) = 0$

c. $A(-2) = -8$ and $A''(3) = 2$

d. $A(-2) = -8$ and $A''(3) = 3$

e. $A(-2) = 8$ and $A''(3) = 2$

(ix) The graph of the function $f(x) = \sin(2x)$ is shown below.



The numerical value of the expression $\int_{0.2}^{0.8} \sin(2x)\, dx$ is equal to the area of the shaded region in the graph above. The value of which of the following expressions is also equal to the area of the shaded region?

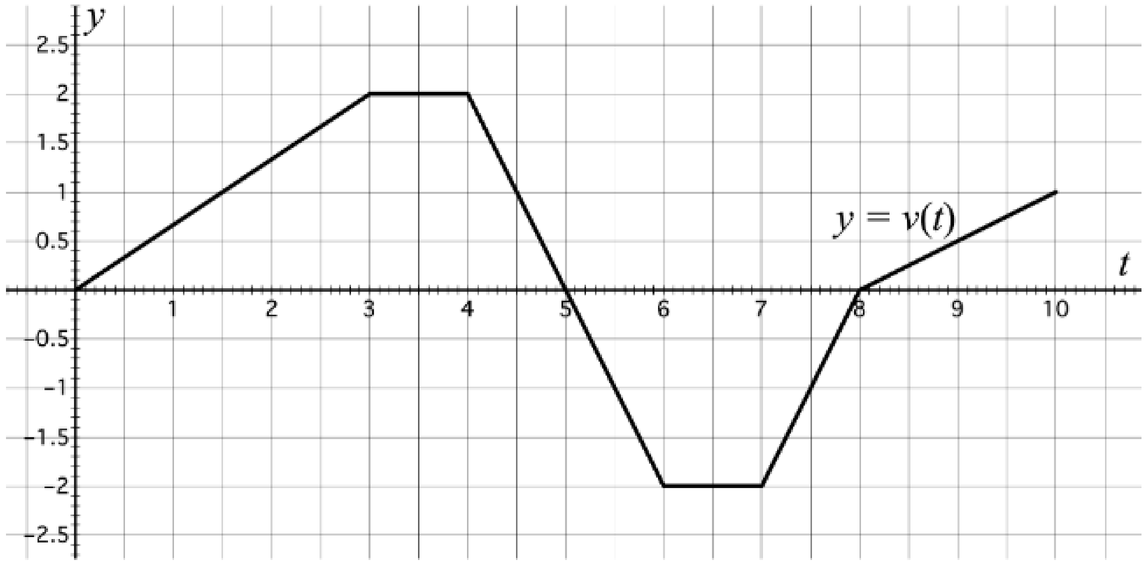
I. $\frac{1}{2} \int_{0.4}^{1.6} \sin(\theta)\, d\theta$

II. $\Delta x \sum_{k=1}^{N} \sin\left(2(0.2 + k\Delta x)\right)$

III. $\frac{1}{2}\left(-\cos(1.6) + \cos(0.4)\right)$

a. I only
b. II only
c. III only
d. I and III only
e. I, II, and III

4. (*3 points each*) The graph of the function $y = v(t)$ is given below. The function v represents the vertical velocity of a car on a roller coaster (in meters per second) in terms of the number of seconds elapsed since the roller coaster started moving. At the beginning of the ride ($t = 0$), the car is 20 meters above the ground. (*Note that the graph is piecewise linear.*)



(a) How high does the car climb in the first 3 seconds of the ride?

(b) What is the car's height at $t = 6$?

(c) For what value of t is the height of the car minimized?

(d) What was the car's acceleration at $t = 9$?

8. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, **do not round**.

(a) $\displaystyle\int_0^{\pi} (1+\cos(x))dx =$

(b) $\displaystyle\int (e^x - 4\sin(x))\, dx =$

(c) $\displaystyle\frac{d}{dx}\int_1^{\ln(x)} \sqrt{2t^4+t+1}dt =$

(d) $\displaystyle\int x^2\sqrt{5+2x^3}dx =$

10. (*8 points*) The function A is given by

$$A(x) = \int_0^{x^2} \left(\sqrt{t} - 10\right)\, dt$$

where x is positive. Determine the coordinates of the point of inflection of the graph of $y = A(x)$. *Write the x and y coordinates in the spaces provided.*

From the Spring 2023 Final Exam

(ii) The expression $\displaystyle\sum_{k=1}^{30} \left(5 + \frac{k}{5}\right)^7 \cdot \frac{1}{5}$ is a Riemann sum approximation of which of the following integrals?

A. $\displaystyle \frac{1}{5}\int_1^{30} \left(5 + \frac{x}{5}\right)^7\, dx$

B. $\displaystyle \int_1^{30} \frac{x^7}{5}\, dx$

C. $\displaystyle \frac{1}{5}\int_1^{30} (5 + x)^7\, dx$

D. $\displaystyle \int_5^{11} x^7\, dx$

E. $\displaystyle \frac{1}{5}\int_5^{11} x^7\, dx$

(iv) Suppose $\int_2^4 f(x)\,dx = 10$ and $\int_2^4 g(x)\,dx = -5$. Which of the following expressions **cannot** be determined from this information alone?

A. $\int_4^2 f(x)\,dx$

B. $\int_2^4 \frac{g(x)}{f(x)}\,dx$

C. $\int_2^4 7g(x)\,dx$

D. $\int_2^4 (f(x) + 5g(x))\,dx$

E. $\int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx$

7. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

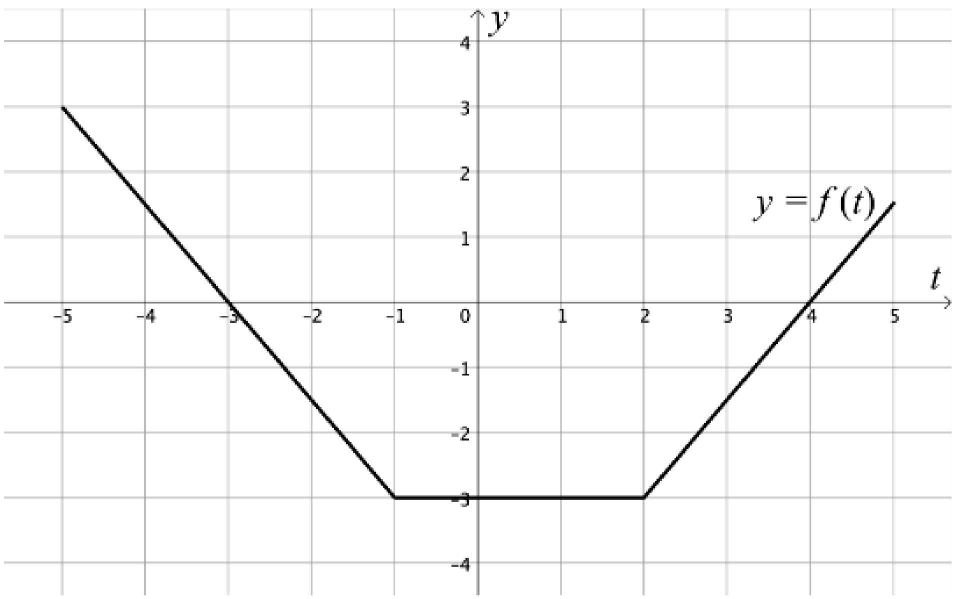
(a) $\int \left(x^5 - \sin(x) + 5^x\right)\,dx =$

(b) $\int_0^{\frac{\pi}{2}} \frac{\cos(\theta)}{1+\sin^2(\theta)}\,d\theta =$

(c) $\dfrac{d}{dx}\displaystyle\int_1^{\sqrt{x}} \ln(4t)\,dt =$

8. (*8 points*) The graph of the function $y = f(t)$ is given below. *Note that the graph of f is defined for all x in the interval* $[-5,\ 5]$. Define $A(x)$ by

$$A(x) = \int_0^x f(t)\,dt.$$



(a) (*4 points*) Evaluate $A(1)$ and $A(-3)$.

(b) (*2 points*) For what value(s) of x on the interval $[-5,\ 5]$ does $A(x)$ have a local maximum?

$x =$ $\boxed{}$

(c) (*2 points*) Find $A'(1)$ and $A''(4)$.

$A'(1) =$ $\boxed{}$

$A''(4) =$ $\boxed{}$

10. (*8 points*) The function A is given by $A(x) = \int_0^x \left(t^2 - 2t\right)\, dt$. Determine the coordinates of the point of inflection of the graph of $y = A(x)$.

Write the x and y coordinates in the spaces provided.

From Fall 2022 Final Exam

(iv) Yvonne decides to go for a run before school. She starts her run from home. The function $y = v(t)$ expresses Yvonne's velocity (in meters per minute) t minutes after she started running. What quantity does the following sum approximate?

$$\sum_{k=1}^{6} v\left(4 + \frac{k-1}{2}\right) \cdot \frac{1}{2}$$

a. The average rate of change of Yvonne's velocity over the interval of time from $t = 1$ to $t = 4$.

b. The change in Yvonne's distance away from home over the interval of time from $t = 4$ to $t = 7$.

c. Yvonne's acceleration over the interval of time from $t = 1$ to $t = 6$.

d. Yvonne's distance away from home after having run for 2.5 minutes.

e. Yvonne's instantaneous velocity 6.5 minutes after having left home.

(v) Suppose $\int_{-2}^{9} f(x)\, dx = 12$ and $\int_{3}^{9} f(x)\, dx = 4$. What is $\int_{-2}^{3} f(x)\, dx$?

a. 16

b. -8

c. 3

d. 8

e. -16

4. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) $\displaystyle\int \left(\sec^2(t) - 9t^2\right) dt =$

(b) $\displaystyle\int_2^3 x \cdot e^{x^2}\, dx =$

(c) $\displaystyle\frac{d}{dx}\int_1^{x^4} \cos(2\theta + 1)\, d\theta =$

7. (*4 points each*) Courtney's velocity (in miles per hour) while running a marathon is given by the function $y = v(t)$, where t represents the number of hours elapsed since Courtney started the marathon.

(a) Write a sentence explaining the meaning of the expression $\displaystyle\int_1^3 v(t)\, dt$.

(b) A marathon is (approximately) 26.2 miles. Given that the integral below equals 26.2, write a sentence explaining the meaning of x in the context of the situation.

$$\int_0^x v(t)\, dt = 26.2$$

From the Spring 2022 Final Exam

(ii) Oil leaks out of a tank at a rate of $r = f(t)$ gallons per minute, where t is measured in minutes. Which of the following expressions represents the **exact** amount of oil that leaked out of the tank from 15 to 45 minutes after oil started leaking?

I. $f(45) - f(15)$

II. $\displaystyle\int_{15}^{45} f(t)\,dt$

III. $\displaystyle\sum_{k=1}^{60} f\left(15 + \frac{k}{2}\right)\cdot\frac{1}{2}$

IV. $\displaystyle\lim_{N\to\infty}\sum_{k=1}^{N} f\left(15 + \frac{30k}{N}\right)\cdot\frac{30}{N}$

a. I only

b. I and II only

c. II and IV only

d. I, II, and IV only

e. I, II, III, and IV

(iv) Let $f(x) > 0$ and $f'(x) > 0$ for $2 \le x \le 4$. Which of the following approximations of $\displaystyle\int_2^4 f(x)\,dx$ is the largest?

a. R_4

b. L_4

c. M_4

d. They are all equal.

e. There is not enough information provided to determine which approximation is largest.

4. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

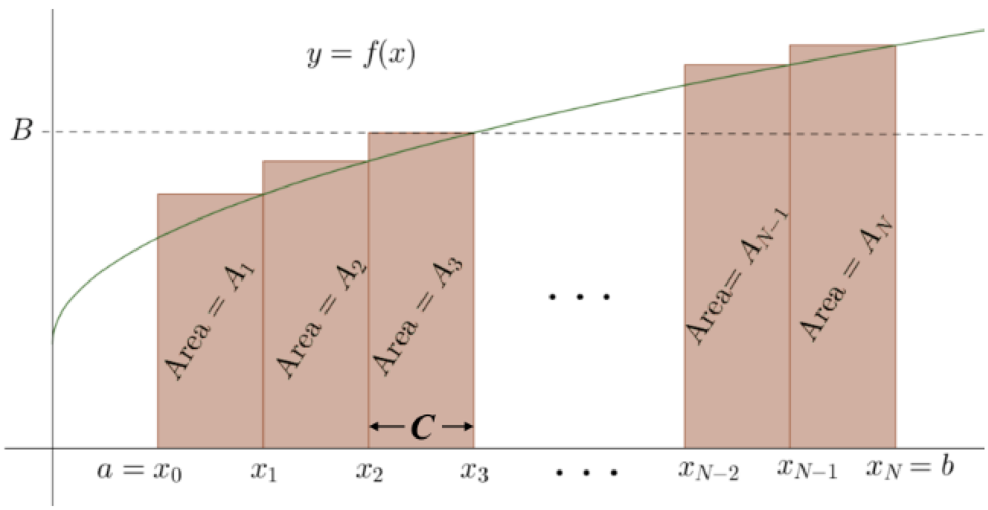
(a) $\displaystyle\int \left(18x^5 - 10x^4 - \frac{28}{x}\right)\,dx =$

(b) $\displaystyle\int_1^3 \frac{dt}{t^2} =$

(c) $\displaystyle\int \theta \sin\left(\theta^2\right)\, d\theta$

(d) $\displaystyle\frac{d}{dx}\int_1^{x^2} \left(t^5 - 9t^3\right)\, dt =$

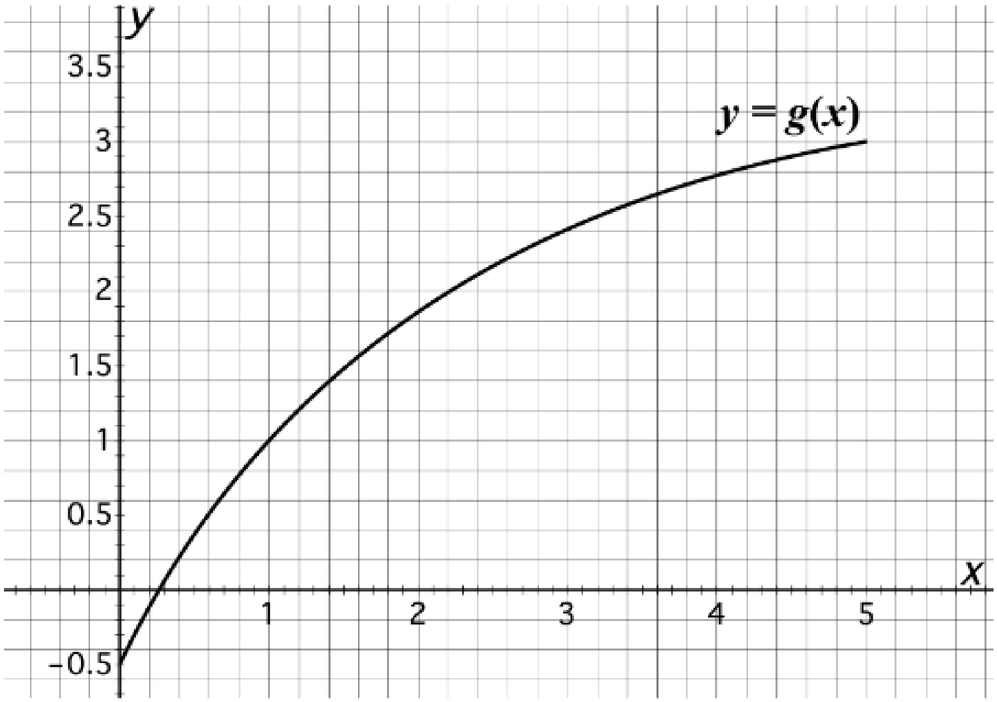
6. (*10 points*) The following image illustrates a Riemann sum using N terms:



Write each item on the right in the blank next to the corresponding expression on the left. Items B, C, and A_3 refer to the labeled graphical quantities above. ***Each item will be used exactly once.***

Expression	Matching Item	Item
Δx	________________	B
$f(x_3)\Delta x$	________________	C
$f(x_3)$	________________	A_3
$\displaystyle\sum_{i=1}^{N} f(x_i)\Delta x$	________________	$\displaystyle\int_a^b f(x)dx$
$\displaystyle\lim_{N\to\infty}\sum_{i=1}^{N} f(x_i)\Delta x$	________________	$A_1 + A_2 + \cdots + A_{N-1} + A_N$

9. (*6 points*) The graph of the function g is given below.



Each of the expressions below represents a numerical value. Identify which of the following expressions represents the largest value and which represents the smallest value. **To receive credit, you must convey your rationale for your selections.**

(i) $g'(1)$

(ii) $\displaystyle\int_1^5 g(x)dx$

(iii) $\displaystyle\sum_{k=1}^{10} g(1+0.4k)\cdot 0.4$

(iv) $g''(1)$

Smallest: ____________________

Largest: ____________________

From the Fall 2021 Final Exam

(iv) The expression $\sum_{k=1}^{8}\left(1+\frac{1}{2}k\right)^3 \cdot \frac{1}{2}$ is a right-endpoint approximation for which of the following integrals?

a. $\frac{1}{2}\int_1^8 x^3\,dx$

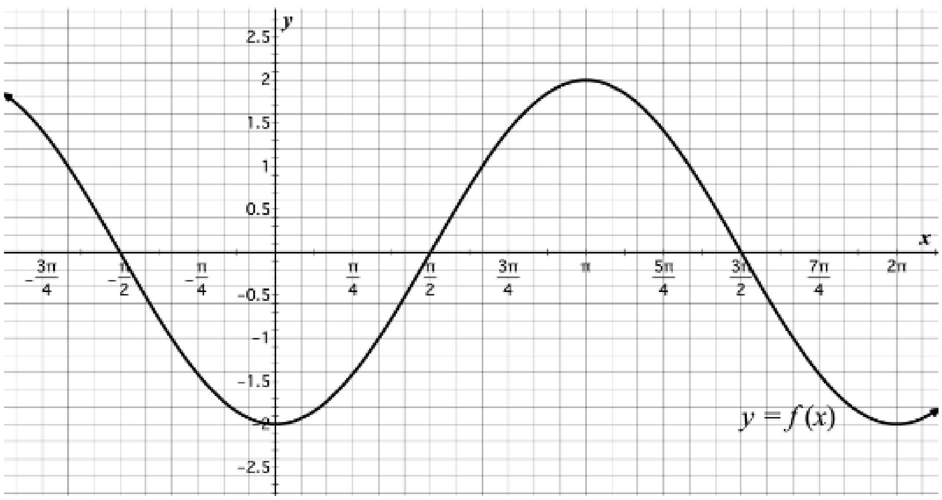
b. $\int_1^5 x^3\,dx$

c. $\frac{1}{2}\int_1^8 \left(1+\frac{1}{2}x\right)^3\,dx$

d. $\int_1^8 (1+x)^3\,dx$

e. $\frac{1}{2}\int_1^5 x^3\,dx$

(v) The graph of the function f is below.



The expressions below represent numerical values. Which expression represents the largest numerical value?

a. $f'\left(\frac{\pi}{2}\right)$

b. $\dfrac{f(\pi)-f\left(\frac{\pi}{2}\right)}{\pi-\frac{\pi}{2}}$

c. $f''(\pi)$

d. $\int_\pi^{2\pi} f(x)\,dx$

e. There is not enough information provided to compare the numerical values that these expressions represent.

4. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

   (a) $\displaystyle\int \left(\frac{1}{x^2} - \cos(x)\right) dx =$

(b) $\displaystyle\int_1^e \frac{\sec^2(\ln(x))}{x}\, dx =$

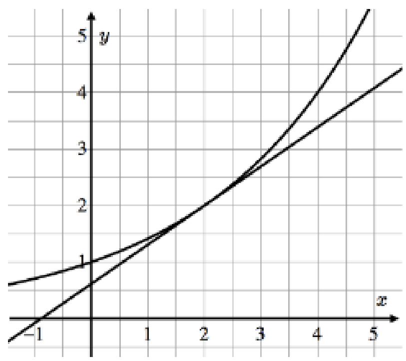
(c) $\displaystyle\frac{d}{dx}\int_\pi^{x^3} \sin\left(4^t\right)\, dt =$

10. (*8 points*) The graphs below are of the function g. Write an expression in each blank that represents the numerical value of the quantity described and identified on the corresponding graph. **Select only from the following answer choices:**

$$\int_1^4 g(x)\,dx \qquad \frac{g(4)-g(1)}{4-1} \qquad g''(2.4) \qquad \sum_{i=1}^{3} g(1+i)$$

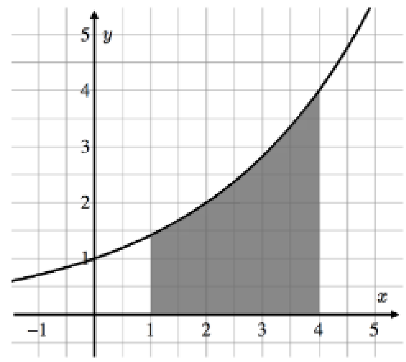
$$\frac{g(4)-g(0)}{4-0} \qquad g'(2) \qquad \int g(x)\,dx \qquad \sum_{i=1}^{N} g(x)\cdot\Delta x$$

(a) Slope of the line tangent to the graph of g at $(2, 2)$.



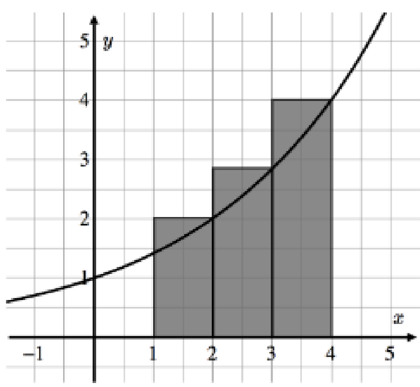
Expression: ______________________

(b) Area bounded by the graph of g and the x-axis on the interval $[1, 4]$.



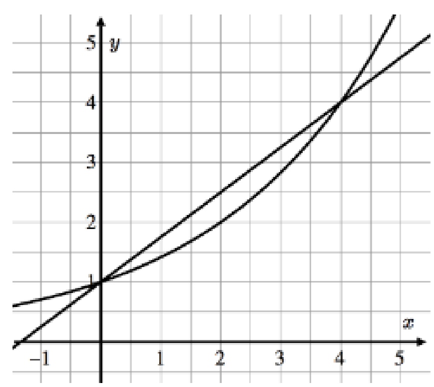
Expression: ______________________

(c) Sum of the area of the rectangles shown below.



Expression: ______________________

(d) Slope of the line secant to the graph of g which passes through the points $(0, 1)$ and $(4, 4)$.



Expression: ______________________

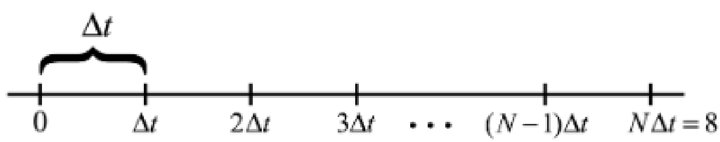
11. (*3 points each*) The efficiency of a car on a road trip can be measured in how slowly fuel is burned. Let $f(t)$ denote the rate

$$f(t) = \text{gallons of fuel burned per minute after } t \text{ minutes on the trip.}$$

The definite integral $\int_0^8 f(t)\,dt$ is approximated by the right-endpoint approximation with N terms:

$$\sum_{i=1}^{N} f(i\Delta t) \cdot \Delta t.$$

(*Note that this sum is based on a uniform partition, as illustrated below.*)



Explain what each of the following expressions represent *in the context of the situation* and specify units of measurement.

(a) $f(3\Delta t) \cdot \Delta t$

(b) $\sum_{i=1}^{N} f(i\Delta t) \cdot \Delta t$

(c) $\lim_{N \to \infty} \sum_{i=1}^{N} f(i\Delta t) \cdot \Delta t$