Radiative Neutrino Mass Models and $(g-2)_\mu$, $R_{K^{(*)}}$, $R_{D^{(*)}}$ Anomalies

Shaikh Saad

Based on: arXiv:2004.07880 (Saad, A. Thapa)
: arXiv:2005.04352 (Saad)

Outline

- Muon anomalous magnetic moment: Δa_μ
- Flavor anomalies: $R_{K^{(*)}}$, $R_{D^{(*)}}$
- Neutrino mass
- Proposals (Model-I, Model-II)
- Summary

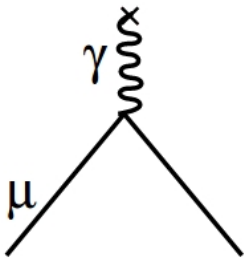
*Talk intended for the graduate students, faculties may find it trivial

$(g-2)_\mu$

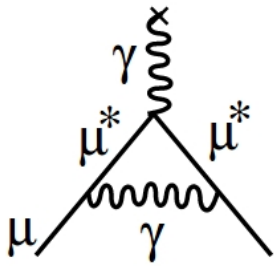
- Dirac's relativistic wave equation formulation: 1928
- Muon magnetic moment: $\vec{M} = g_\mu \frac{e}{2m_\mu} \vec{S}$
- Landé g-factor: $g_\mu = 2$

$(g-2)_\mu$

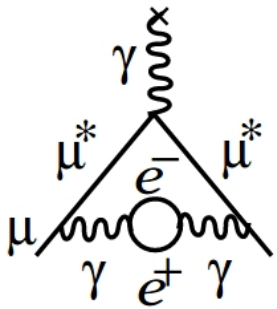
- Quantum loop corrections: $g_\mu \neq 2$



Dirac



Schwinger

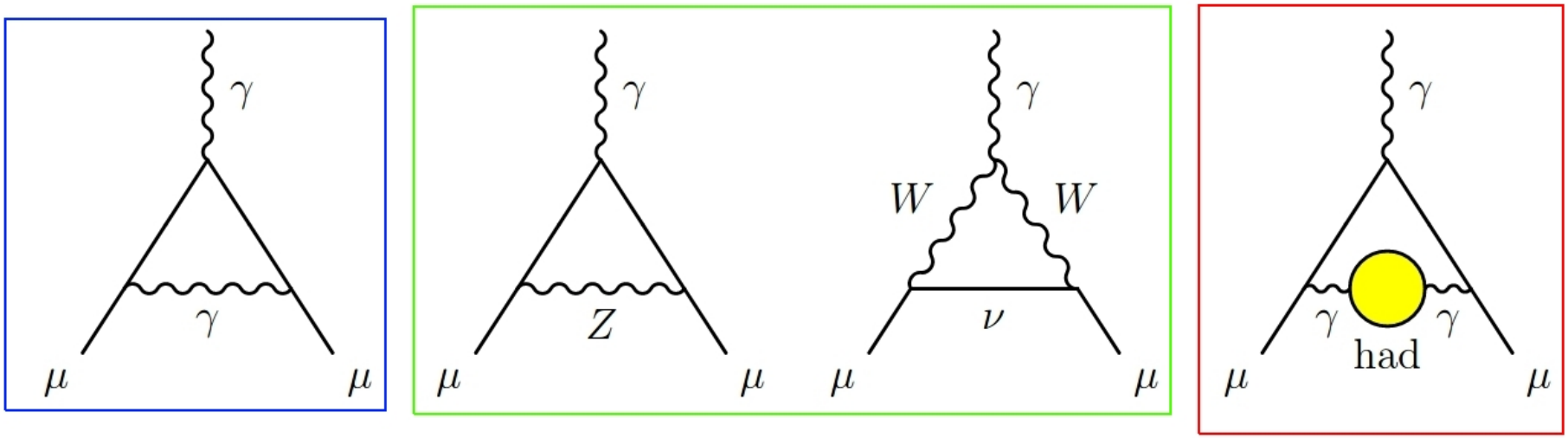


Bethe (1947) did before Schwinger (1948), but in non-relativistic framework

- Anomalous magnetic moment: $a_\mu = \frac{g_\mu - 2}{2}$

$(g-2)_\mu$

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm Had}$$



$$a_\mu^{\rm SM} = 116\,591\,830(1)(40)(26) \times 10^{-11}$$

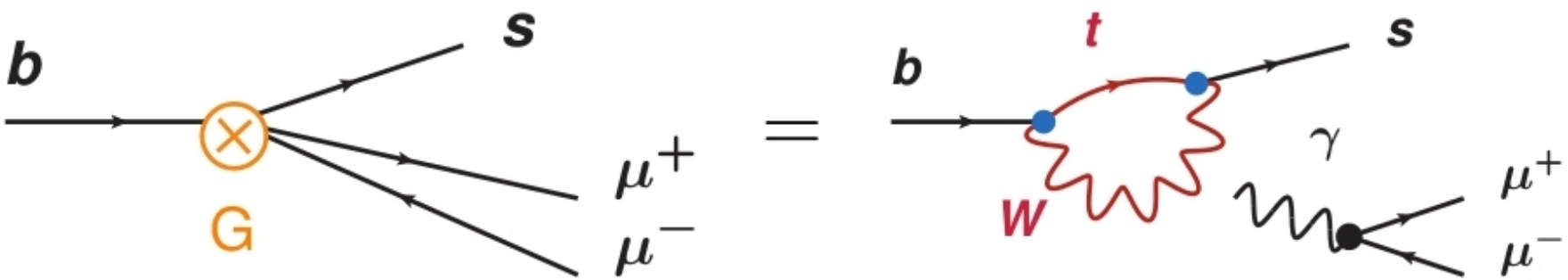
errors: electroweak, lowest-order hadronic, and higher-order hadronic contributions

- $a_\mu^{exp} - a_\mu^{SM} \equiv \Delta a_\mu = (274 \pm 73) \times 10^{-11}$ $\sim 3.7\sigma$

$R_{K^{(*)}}$

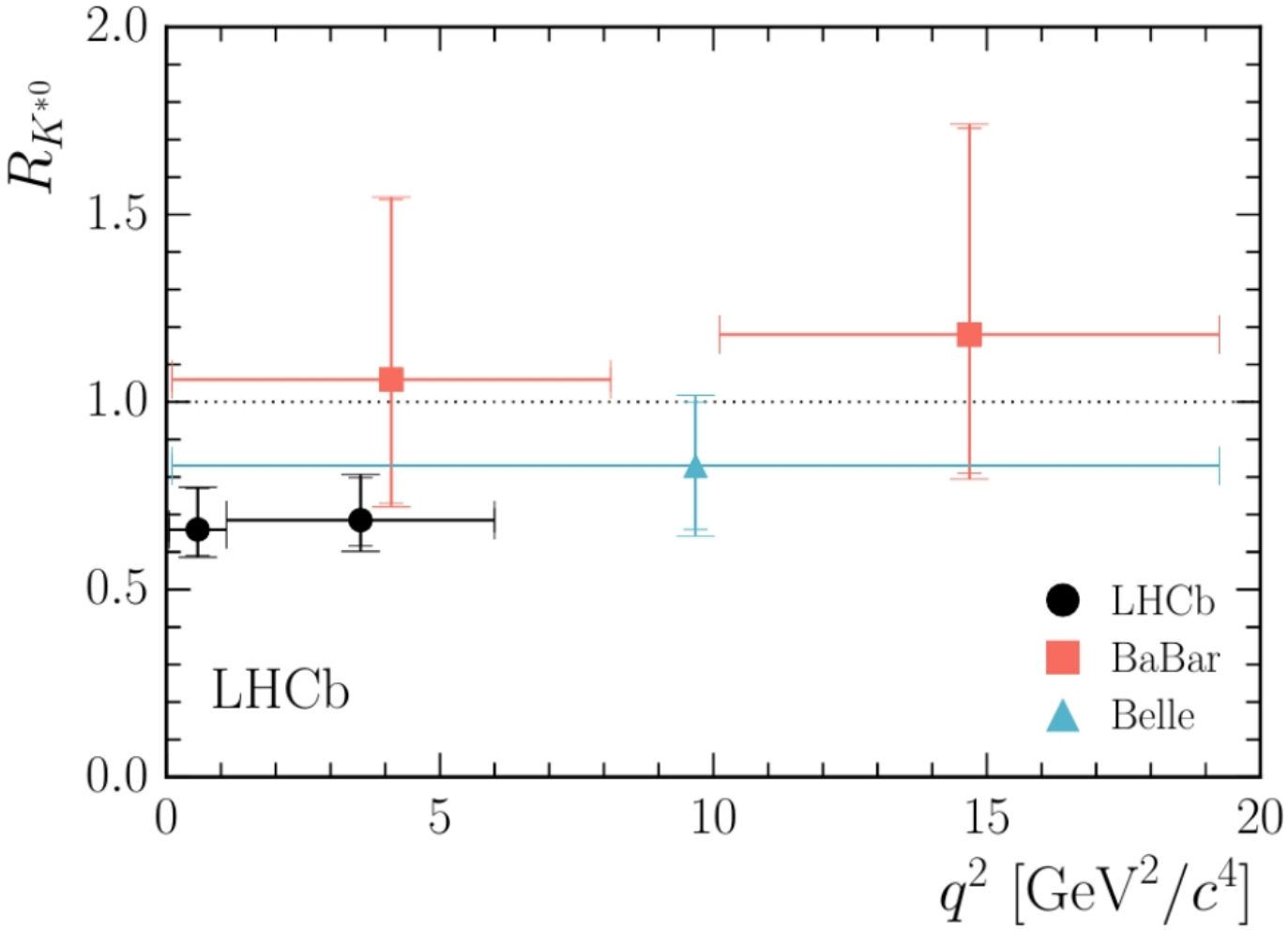
- $b \to s$: Neutral current process

$$R_K = \frac{\Gamma(\overline{B} \to \overline{K}\mu^+\mu^-)}{\Gamma(\overline{B} \to \overline{K}e^+e^-)}, \quad R_{K^*} = \frac{\Gamma(\overline{B} \to \overline{K}^*\mu^+\mu^-)}{\Gamma(\overline{B} \to \overline{K}^*e^+e^-)}.$$



$$R_K^{\text{SM}} = 1.0003 \pm 0.0001, \quad R_{K^*}^{\text{SM}} = 1.00 \pm 0.01$$

$R_{K^{(*)}}$



LHCb, arXiv:1705.05802 [hep-ex]

$R_{K^{(*)}}$

- LHCb, arXiv:1903.09252

$$R_K^{\text{exp}} = 0.846^{+0.06+0.016}_{-0.054-0.014}, \quad 1.1\ \text{GeV}^2 < q^2 < 6.0\ \text{GeV}^2$$

- Belle, arXiv:1904.02440

$$R_{K^*}^{\text{exp}} = \begin{cases} 0.90^{+0.27}_{-0.21} \pm 0.10, & 0.1\ \text{GeV}^2 < q^2 < 8.0\ \text{GeV}^2 \\ 1.18^{+0.52}_{-0.32} \pm 0.10, & 15\ \text{GeV}^2 < q^2 < 19\ \text{GeV}^2 \end{cases}$$

- LHCb, arXiv:1705.05802

$$R_{K^*}^{\text{exp}} = \begin{cases} 0.660^{+0.110}_{-0.070} \pm 0.024, & 0.045\ \text{GeV}^2 < q^2 < 1.1\ \text{GeV}^2 \\ 0.685^{+0.113}_{-0.069} \pm 0.047, & 1.1\ \text{GeV}^2 < q^2 < 6.0\ \text{GeV}^2 \end{cases}$$

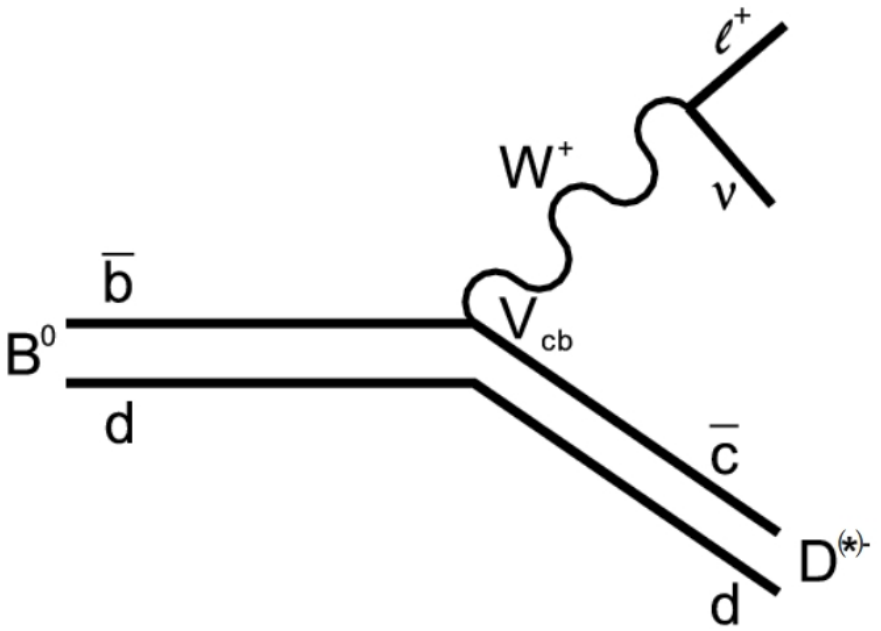
$R_{K^{(*)}}$

- R_K $\sim 2.5\sigma$
- R_{K^*} $\sim 2.5\sigma$ (LHCb)
- Angular observables: $P_4^{'}, P_5^{'}$
- $B_s \to \mu\mu$ (ATLAS, CMS, LHCb)
- and more ...
- Combined: $\sim 4.5\sigma$

$R_{D(*)}$

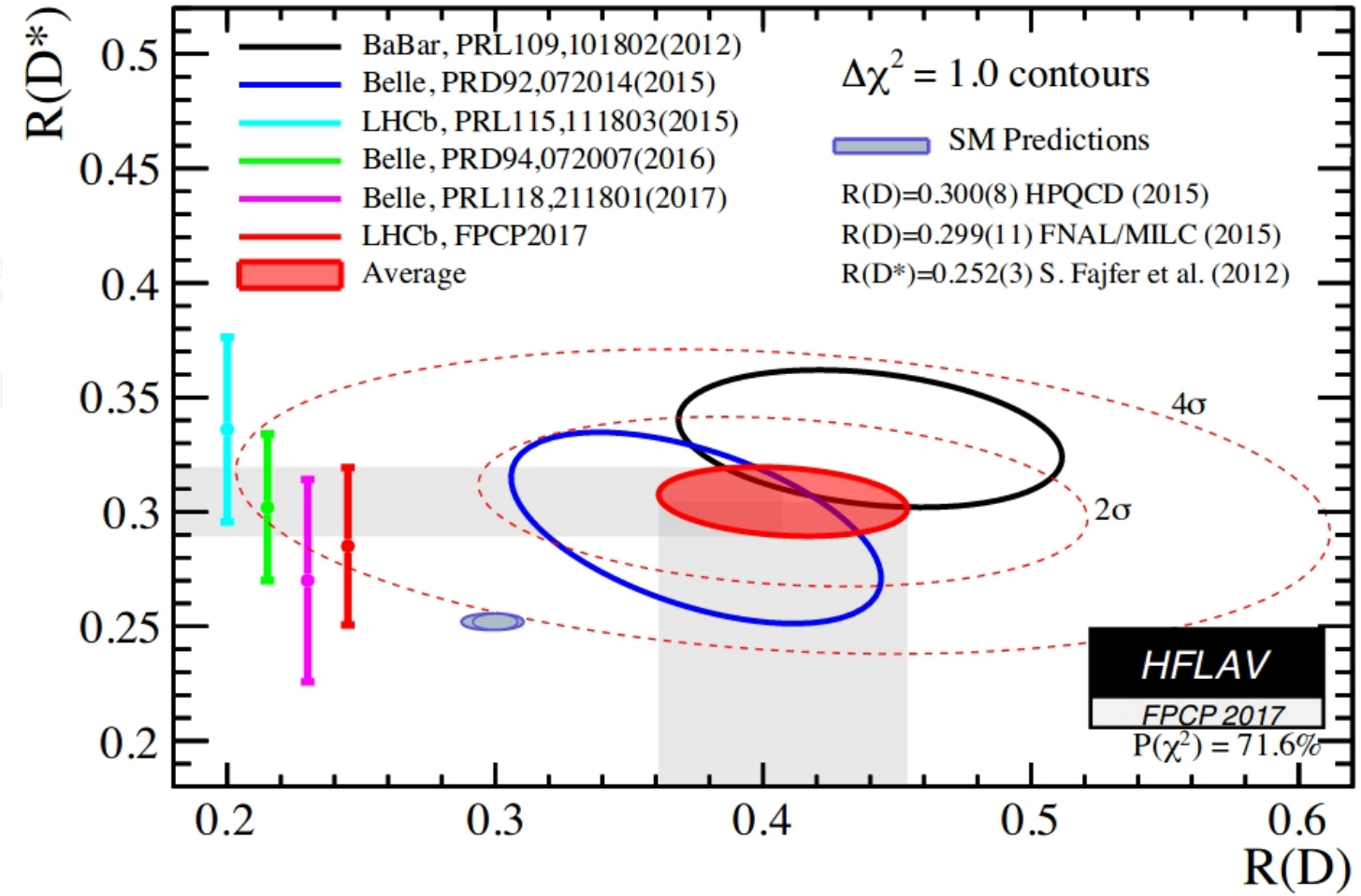
- $b \to c$: Charged current process

$$R_D = \frac{\Gamma(\overline{B} \to D\tau\nu)}{\Gamma(\overline{B} \to D\ell\nu)}, \quad R_{D^*} = \frac{\Gamma(\overline{B} \to D^*\tau\nu)}{\Gamma(\overline{B} \to D^*\ell\nu)}$$



$$R_D^{\text{SM}} = 0.299 \pm 0.003, \quad R_{D^*}^{\text{SM}} = 0.258 \pm 0.005$$

$R_{D(*)}$


BaBar, PRL109,101802(2012)
Belle, PRD92,072014(2015)
LHCb, PRL115,111803(2015)
Belle, PRD94,072007(2016)
Belle, PRL118,211801(2017)
LHCb, FPCP2017
Average
$\Delta\chi^2 = 1.0$ contours
SM Predictions
R(D)=0.300(8) HPQCD (2015)
R(D)=0.299(11) FNAL/MILC (2015)
R(D*)=0.252(3) S. Fajfer et al. (2012)
4σ
2σ
HFLAV
FPCP 2017
$P(\chi^2) = 71.6\%$
R(D*)
R(D)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.2
0.3
0.4
0.5
0.6

$R_{D(*)}$

- Global average: (Belle, BaBar, LHCb)

$$R_D^{\text{exp}} = 0.334 \pm 0.031, \quad R_{D^*}^{\text{exp}} = 0.297 \pm 0.015$$

- R_D, R_{D^*} $\sim 3\sigma$
- $R_{J/\psi}$, $f_L^{D^*}$, $\mathcal{P}_\tau^8$
- and more ...

Neutrino Mass

- In SM, neutrino mass $= 0$

Oscillation parameters	$1\ \sigma$ allowed range from NuFit4.1	$3\ \sigma$ allowed range from NuFit4.1
$\sin^2\theta_{12}$	$0.31^{+0.013}_{-0.012}$	0.275 - 0.350
$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	0.02046 - 0.02440
$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	0.427 - 0.609
$\Delta m^2_{21}\ (10^{-5}eV^2)$	$7.39^{+0.21}_{-0.20}$	6.79 - 8.01
$\Delta m^2_{31}\ (10^{-3}eV^2)$	$2.523^{+0.032}_{-0.030}$	2.432 - 2.618

Synopsis

- SM cannot explain neutrino oscillation data
- Long-standing tension in $(g-2)_\mu$
- Large deviations in flavor ratios

Solution?

✗ Standard Model

✓ Physics Beyond the Standard Model

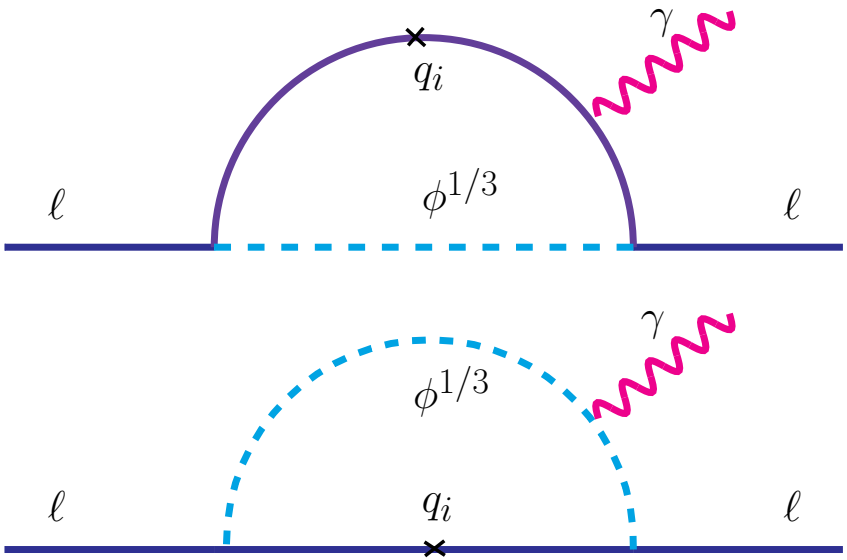
- Can all these be related? Combined explanations?

✓ Scalar Leptoquarks

➳ Which Leptoquarks?

LQ Solution: $(g-2)_\mu$

✓ $S_1 \sim (\overline{3}, 1, 1/3)$



$$\Delta a_\mu \simeq -\frac{3}{8\pi^2}\frac{m_t m_\mu}{M_1^2} y_{32}^L y_{32}^R \left[\frac{7}{6} + \frac{2}{3}\log[x_t]\right].$$

✓ $R_2 \sim (3, 2, 7/6)$

LQ Solution: Flavor anomalies

arXiv:1808.08179

Model	$R_{K^{(*)}}$	$R_{D^{(*)}}$	$R_{K^{(*)}}$ & $R_{D^{(*)}}$
S_1	✗*	✓	✗*
R_2	✗*	✓	✗
$\widetilde{R_2}$	✗	✗	✗
S_3	✓	✗	✗
U_1	✓	✓	✓
U_3	✓	✗	✗

✗ Single LQ

Two possibilities

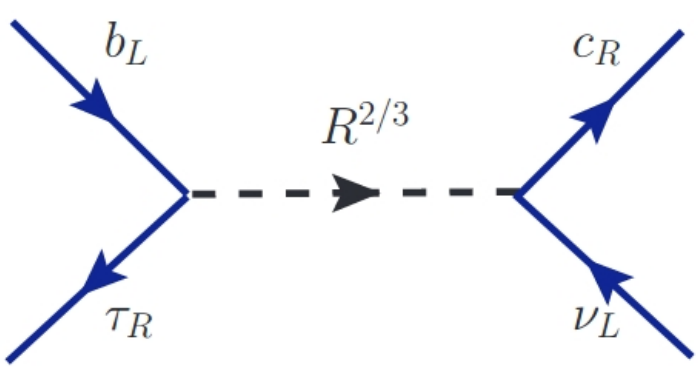
Model	$R_{K^{(*)}}$	$R_{D^{(*)}}$	$R_{K^{(*)}}$ & $R_{D^{(*)}}$
S_1	✗*	✓	✗*
R_2	✗*	✓	✗
$\widetilde{R_2}$	✗	✗	✗
S_3	✓	✗	✗

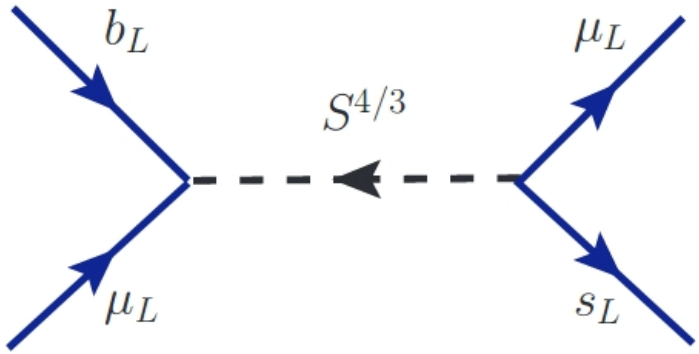
✓ $R_2 \sim (3, 2, 7/6)$ + $S_3 \sim (\overline{3}, 3, 1/3)$: Model-I

✓ $S_1 \sim (\overline{3}, 1, 1/3)$ + $S_3 \sim (\overline{3}, 3, 1/3)$: Model-II

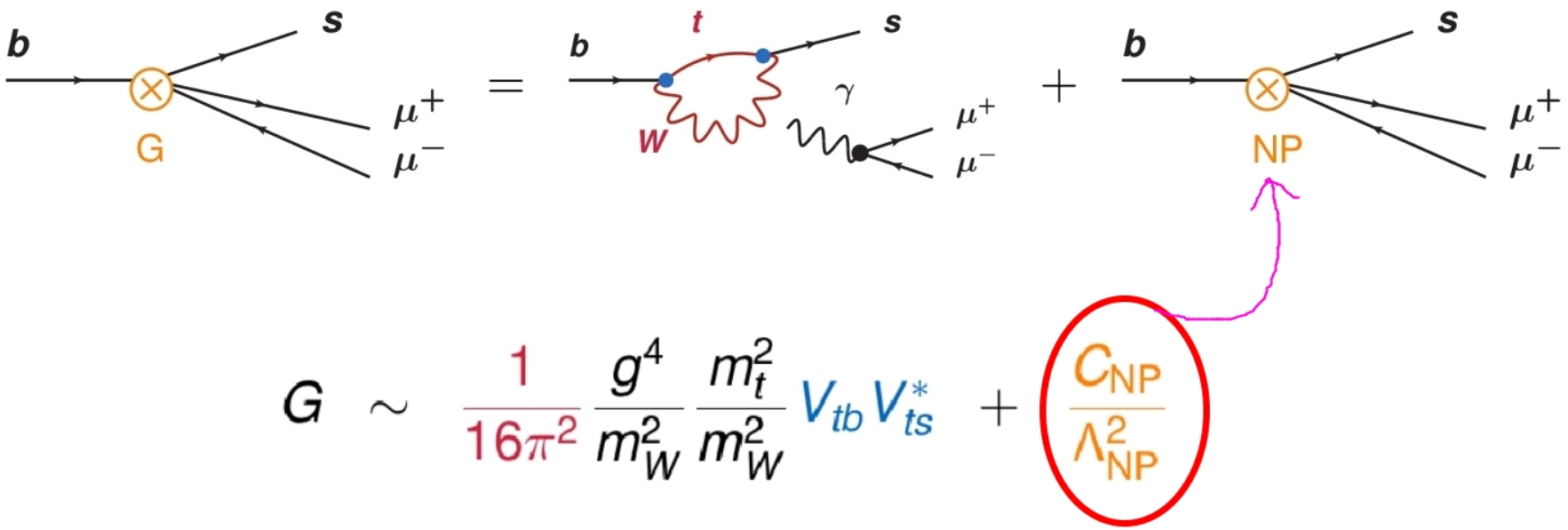
Model-I

$R_2 \sim (3, 2, 7/6)$ + $S_3 \sim (\overline{3}, 3, 1/3)$





$R_{K^{(*)}}$



$$G \sim \frac{1}{16\pi^2}\frac{g^4}{m_W^2}\frac{m_t^2}{m_W^2}V_{tb}V_{ts}^* + \frac{C_{NP}}{\Lambda_{NP}^2}$$

Wolfgang

$S_3 \sim (\bar{3}, 3, 1/3)$

- $C_9^{\mu\mu} = -C_{10}^{\mu\mu} = -0.53$

arXiv:1903.10434

$R_{K^{(*)}}$

$S_3 \sim (\overline{3}, 3, 1/3)$

$$\mathcal{H}_{eff}^{dd\ell\ell} = -\frac{4G_F}{\sqrt{2}} V_{tj} V_{ti}^* \left(\sum_{X=9,10} C_X^{ij,\ell\ell'} \mathcal{O}_X^{ij,\ell\ell'} \right) + h.c.,$$

$$\mathcal{O}_9^{ij,\ell\ell'} = \frac{\alpha}{4\pi} \left(\overline{d}_i \gamma^\mu P_L d_j\right) \left(\overline{\ell} \gamma_\mu \ell'\right), \; \mathcal{O}_{10}^{ij,\ell\ell'} = \frac{\alpha}{4\pi} \left(\overline{d}_i \gamma^\mu P_L d_j\right) \left(\overline{\ell} \gamma_\mu \gamma_5 \ell'\right).$$

$$C_9^{\ell\ell'} = -C_{10}^{\ell\ell'} = \frac{v^2}{V_{tb} V_{ts}^*} \frac{\pi}{\alpha_{em}} \frac{y_{b\ell'}^S \left(y_{s\ell}^S\right)^*}{M_3^2}.$$

- $C_9^{\mu\mu} = -C_{10}^{\mu\mu} = -0.53$ arXiv:1903.10434

$R_{D^{(*)}}$

$R_2 \sim (3, 2, 7/6)$

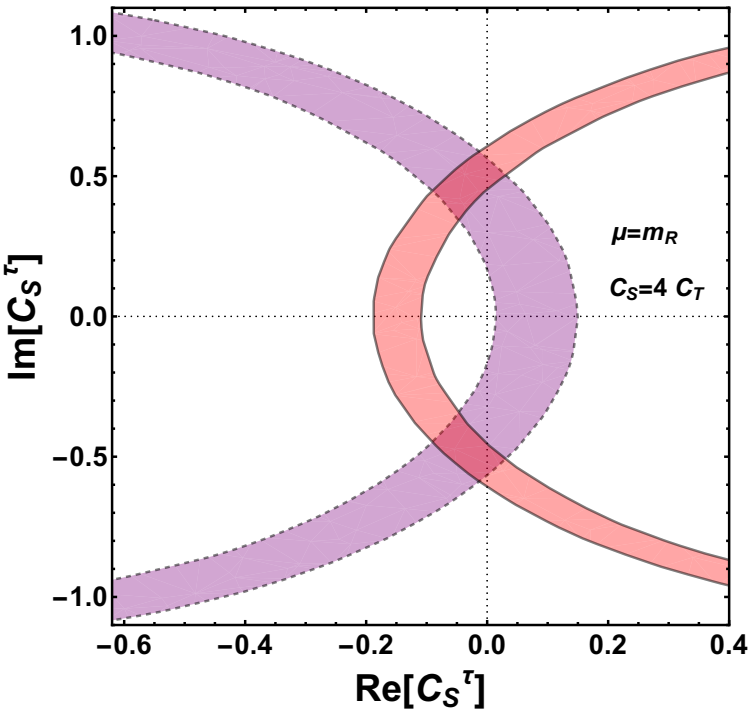
$$\mathcal{H}_{\text{eff}}^{du\ell\nu} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[C_V^{fi} \left(\overline{\ell}_L \gamma^\mu \nu_{Li} \right) \left(\overline{c}_L \gamma_\mu b_L \right) + C_S^{fi} \left(\overline{\ell}_{Rf} \nu_{Lj} \right) \left(\overline{c}_R b_L \right) \right.$$
$$\left. + C_T^{fi} \left(\overline{\ell}_{Rf} \sigma^{\mu\nu} \nu_{Li} \right) \left(\overline{c}_R \sigma_{\mu\nu} b_L \right) \right] +$$

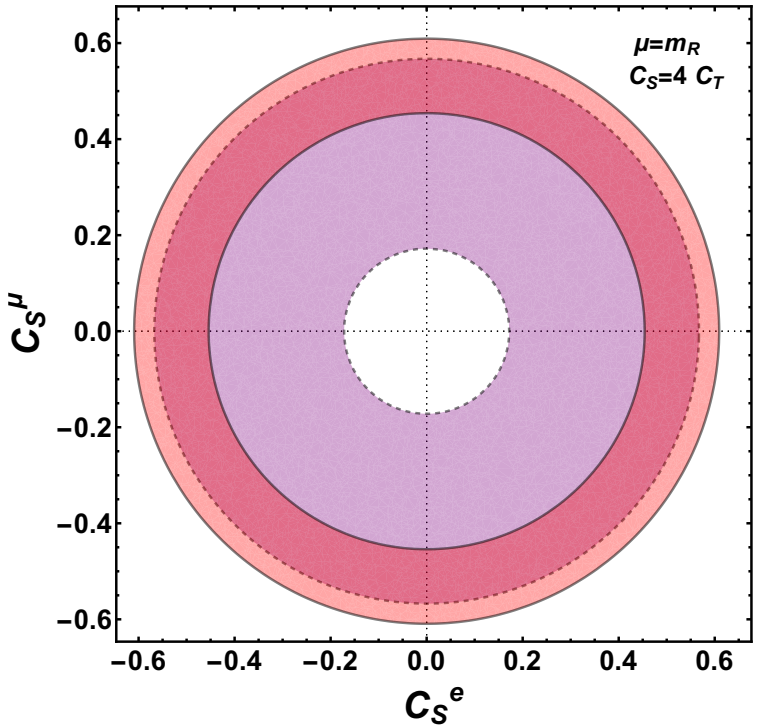
$$C_S^j \left(\mu = m_R \right) = 4 C_T^j \left(\mu = m_R \right) = \frac{\hat{y}_{cj}^L (y_{b\tau}^R)^*}{4\sqrt{2} m_R^2 G_F V_{cb}}.$$

- $C_S = 4 C_T$

$R_{D(*)}$

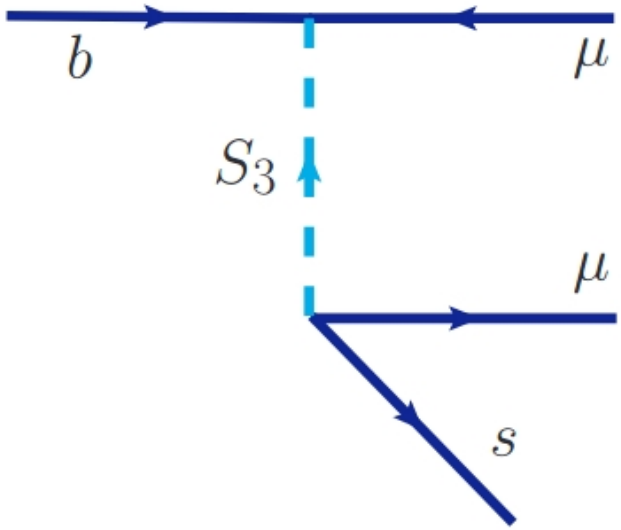
$R_2 \sim (3, 2, 7/6)$: $C_S = 4C_T$

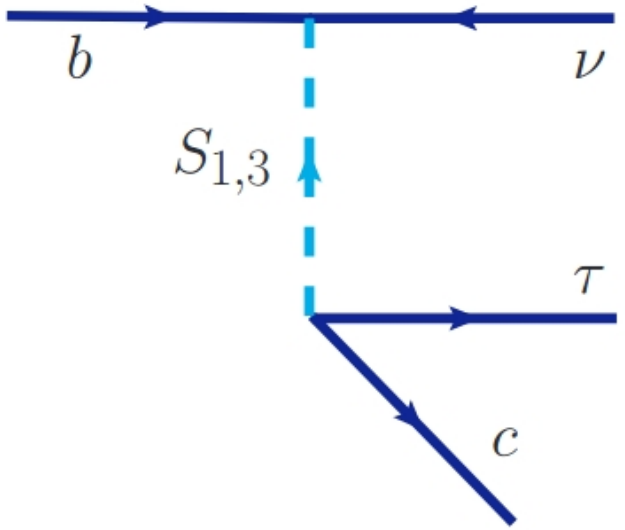




Model-II

$S_1 \sim (\overline{3}, 1, 1/3)$ + $S_3 \sim (\overline{3}, 3, 1/3)$





$R_{D^{(*)}}$

$S_3 \sim (\overline{3}, 3, 1/3)$ + $S_1 \sim (\overline{3}, 1, 1/3)$

$$\mathcal{H}_{\text{eff}}^{du\ell\nu} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[C_V^{fi} \left(\overline{\ell}_L \gamma^\mu \nu_{Li} \right) \left(\overline{c}_L \gamma_\mu b_L \right) + C_S^{fi} \left(\overline{\ell}_{Rf} \nu_{Lj} \right) \left(\overline{c}_R b_L \right) \right.$$
$$\left. + C_T^{fi} \left(\overline{\ell}_{Rf} \sigma^{\mu\nu} \nu_{Li} \right) \left(\overline{c}_R \sigma_{\mu\nu} b_L \right) \right] +$$

$$C_S^{fi} = -4 C_T^{fi} = -\frac{v^2}{4 V_{cb}} \frac{y_{bi}^L \left(y^R \right)_{cf}^*}{M_1^2},$$
$$C_V^{fi} = \frac{v^2}{4 V_{cb}} \left[\frac{y_{bi}^L \left(V^* y^L \right)_{cf}^*}{M_1^2} - \frac{y_{bi}^S \left(V^* y^S \right)_{cf}^*}{M_3^2} \right].$$

- $C_S = -4 C_T$

Neutrino mass?

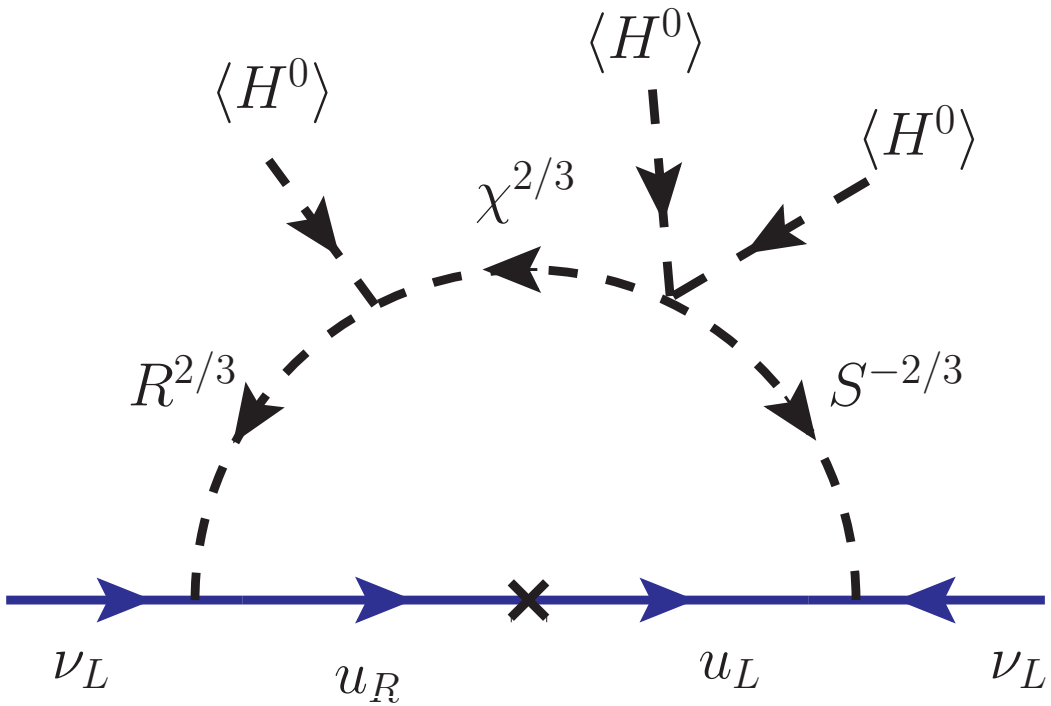
✗ $R_2 \sim (3, 2, 7/6)$ + $S_3 \sim (\overline{3}, 3, 1/3)$: Model-I

✗ $S_1 \sim (\overline{3}, 1, 1/3)$ + $S_3 \sim (\overline{3}, 3, 1/3)$: Model-II

- Minimal choice: single scalar

Model-I: Neutrino mass

$R_2 \sim (3, 2, 7/6)$ + $S_3 \sim (\bar{3}, 3, 1/3)$ + $\chi_1 \sim (3, 1, 2/3)$



arXiv: 1907.09498

$$\mathcal{M}^{\nu}_{ij} = m_0 \left[(y^L)^*_{ki} m^u_k (V^* y)_{kj} + (i \leftrightarrow j) \right]; \quad m_0 \approx \frac{1}{16\pi^2} \frac{\mu \lambda v^3}{M_1^2 M_2^2}$$

Model-I: Combined explanations

TX-I :

$$y^R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}, \quad y^L = \begin{pmatrix} 0 & 0 & 0 \\ * & * & * \\ 0 & 0 & * \end{pmatrix}, \quad y = \begin{pmatrix} 0 & 0 & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix}.$$

TX-II :

$$y^R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}, \quad y^L = \begin{pmatrix} 0 & 0 & 0 \\ * & * & * \\ 0 & 0 & * \end{pmatrix}, \quad y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ * & * & 0 \end{pmatrix}.$$

Model-I

5 Experimental Constraints

LHC bounds

Decays	Scalar LQ (GeV)			$\mathcal{L}_{\text{int}}$
	$\beta = 1$	$\beta = 0.8$	$\beta = 0.5$	
$jj\nu\bar{\nu}$	980	810	635	$35.9\,\text{fb}^{-1}$
$jje\bar{e}$	1435	1360	1200	$35.9\,\text{fb}^{-1}$
$jj\mu\bar{\mu}$	1525	1445	1280	$36.1\,\text{fb}^{-1}$
$t\bar{t}\tau\bar{\tau}$	930	850	730	$36.1\,\text{fb}^{-1}$
$t\bar{t}\nu\bar{\nu}$	1020	940	812	$35.9\,\text{fb}^{-1}$
$b\bar{b}\tau\bar{\tau}$	1025	965	835	$36.1\,\text{fb}^{-1}$

LHC (ATLAS, CMS)

Model-I: BM-TX-I

$M_2 = 1.2$ TeV, $M_3 = 2.5$ TeV

$$y^R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1.09527\, i \end{pmatrix},$$

$$y^L = \begin{pmatrix} 0 & 0 & 0 \\ 4.0503 \times 10^{-3} & -2.1393 \times 10^{-2} & 1.2243 \\ 0 & 0 & -4.9097 \times 10^{-4} \end{pmatrix},$$

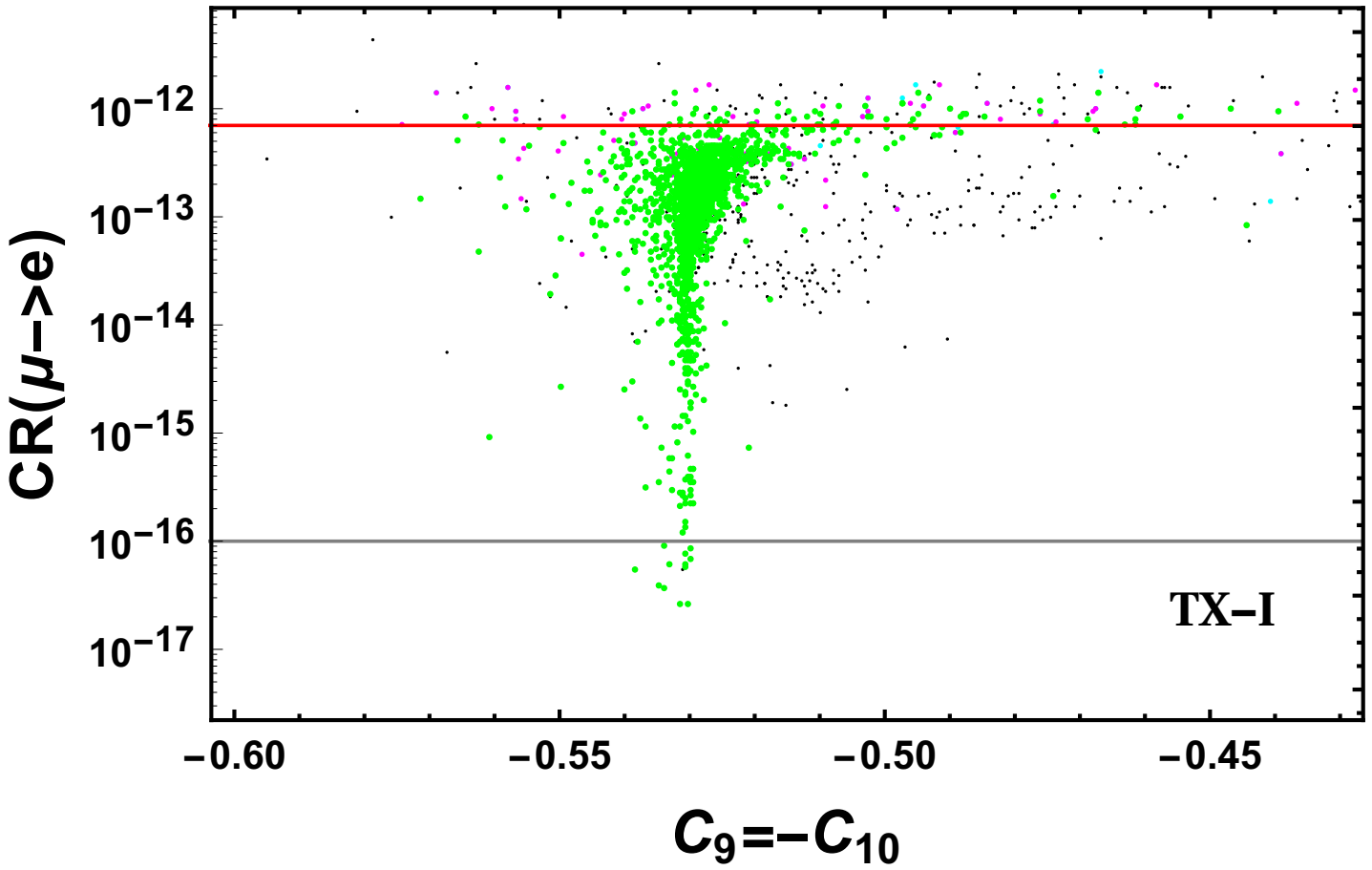
$$y = \begin{pmatrix} 0 & 0 & 0 \\ -4.5241 \times 10^{-4} & -7.7187 \times 10^{-3} & -4.6354 \times 10^{-4} \\ 0 & 6.8578 \times 10^{-1} & 0 \end{pmatrix},$$

$m_0 = 1.297 \times 10^{-8}$.

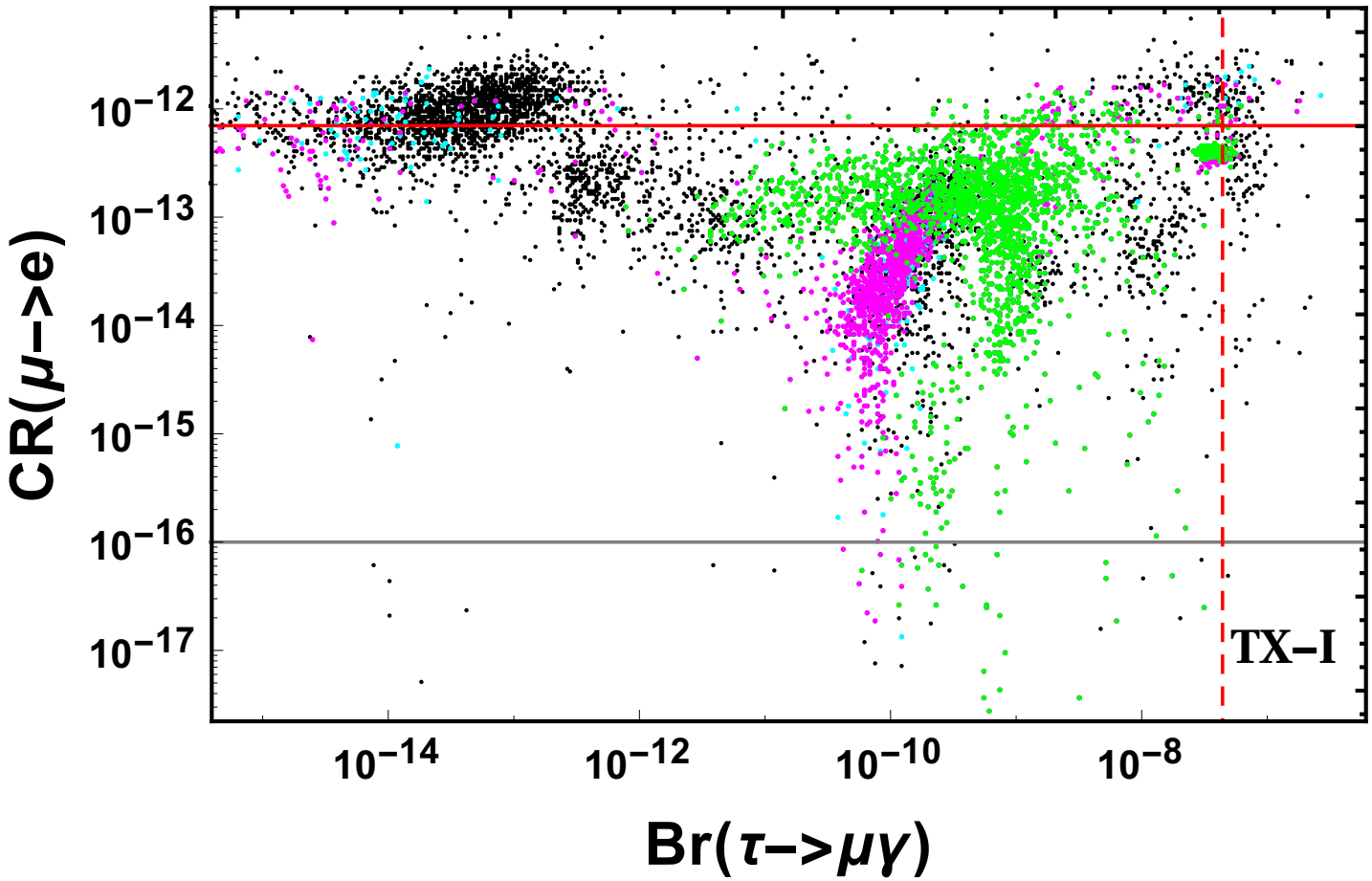
Model-I: BM fits

Observables	BM-TX-I	BM-TX-II
$\Delta m^2_{21}(eV^2)$	7.39×10^{-5}	7.40×10^{-5}
$\Delta m^2_{31}(eV^2)$	2.52×10^{-3}	2.52×10^{-3}
θ_{12}	33.86°	33.87°
θ_{23}	47.95°	49.95°
θ_{13}	8.61°	8.61°
R_D	0.3485	0.3484
R_{D^*}	0.2875	0.2875
$C_9 = -C_{10}$	-0.529	-0.530
$CR(\mu \to e)$	3.31×10^{-15}	5.90×10^{-17}
$Br(\mu \to e\gamma)$	1.34×10^{-14}	1.45×10^{-14}
$Br(\tau \to e\gamma)$	4.11×10^{-10}	1.49×10^{-11}
$Br(\tau \to \mu\gamma)$	1.47×10^{-11}	4.95×10^{-10}

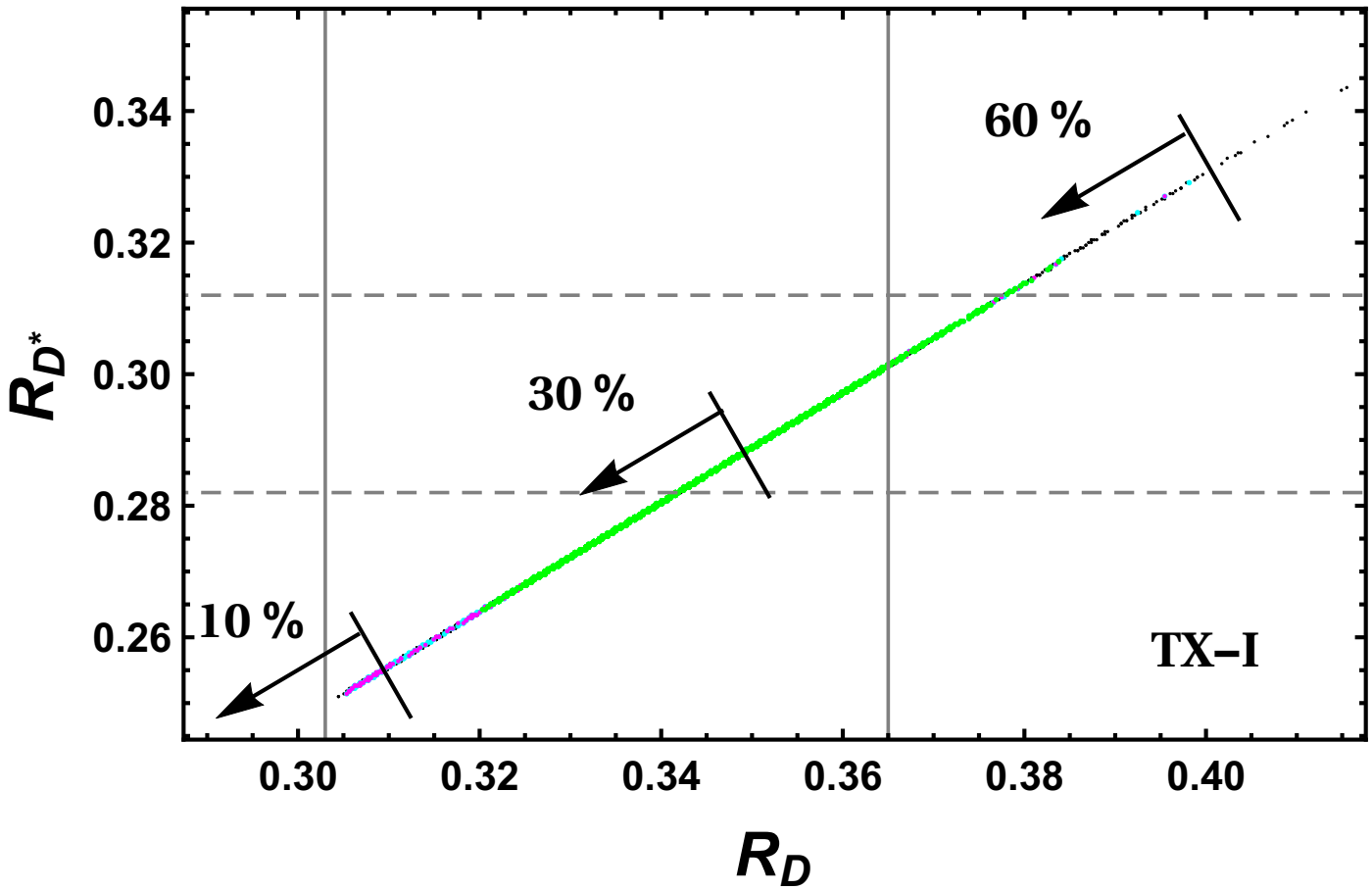
Model-I



Model-I


CR(μ–>e)
10^−12
10^−13
10^−14
10^−15
10^−16
10^−17
TX–I
10^−14
10^−12
10^−10
10^−8
Br(τ–>μγ)

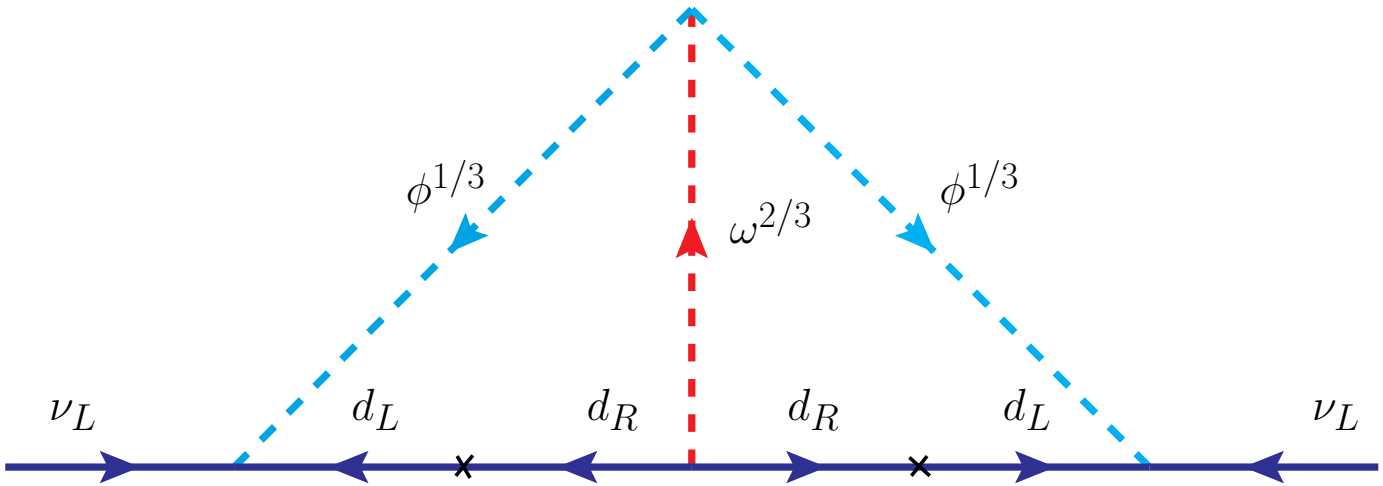
Model-I



$Br(B_c \to \tau\nu)\%$

Model-II: Neutrino mass

$S_1 \sim (\overline{3}, 1, 1/3)$ + $S_3 \sim (\overline{3}, 3, 1/3)$ + $\omega \sim (\overline{6}, 1, 2/3)$



Babu, Leung 2001

$$\mathcal{M}^{\nu}_{ij} = 24\mu_p\, y^p_{li}\, m^d_{ll}\, y^{\omega}_{lk}\, \mathcal{I}^p_{lk}\, m^d_{kk}\, y^p_{kj}; \quad \mathcal{I}^p_{lk} = \frac{1}{256\pi^4}\frac{1}{M_p^2}\overline{\mathcal{I}}\left[\frac{M^2_{DQ}}{M_p^2}\right]$$

Model-II: Combined explanations

$M_{1,3} = 1.2$ TeV

$$y^R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix}, \; y^L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix},$$

$$y^S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ * & * & * \end{pmatrix}, \; y^\omega = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}.$$

Model-II

4 Correlated Observables

Model-II: BM-I

$$y^L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.09485 & -1.413 \\ 0 & 0.01699 & -0.05935 \end{pmatrix},$$

$$y^R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1.451 \\ 0 & 0.1900 & 0 \end{pmatrix},$$

$$y^S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.03230 & -0.4183 \\ 0.002867 & 0.03398 & 0.1742 \end{pmatrix},$$

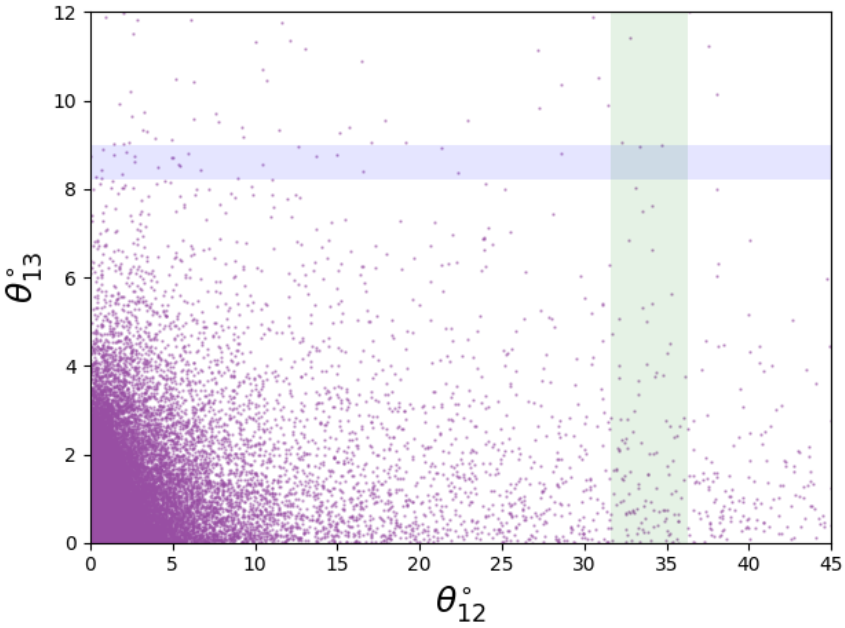
$$y^\omega = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1.451 & 0.1332 \\ 0 & 0.1332 & 0.04726 \end{pmatrix}.$$

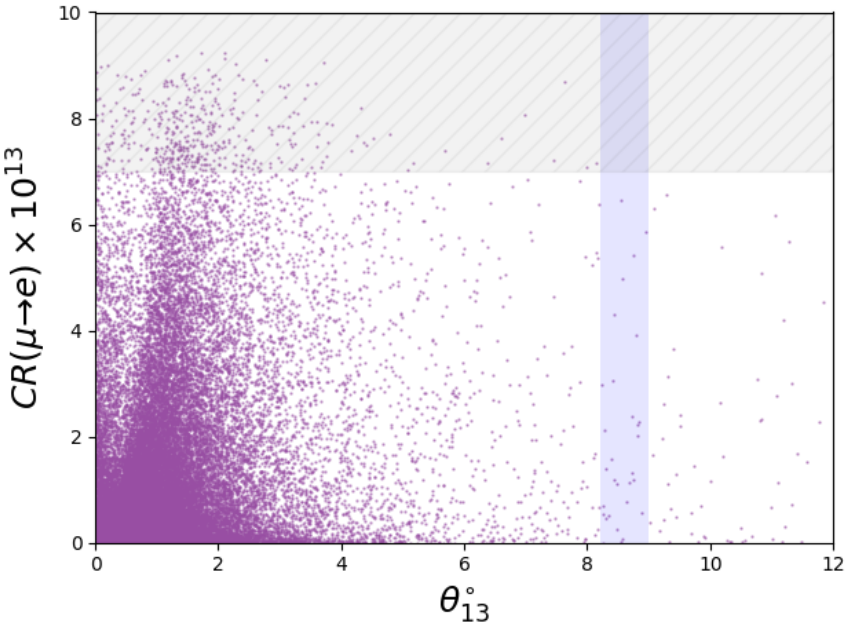
Model-II

Observables	BM-I	BM-II
$\Delta m^2_{21}(eV^2)$	7.348×10^{-5}	7.383×10^{-5}
$\Delta m^2_{31}(eV^2)$	2.526×10^{-3}	2.524×10^{-3}
θ_{12}	33.698°	33.813°
θ_{23}	48.373°	48.055°
θ_{13}	8.624°	8.615°
$(g-2)_\mu$	2.718×10^{-9}	2.688×10^{-9}
$C_9 = -C_{10}$	-0.528	-0.530
R_D	0.339	0.340
R_{D^*}	0.286	0.285
$Br(\mu \to e\gamma)$	8.284×10^{-15}	2.740×10^{-16}
$CR(\mu \to e)$	3.243×10^{-16}	1.377×10^{-16}
$Br(\tau \to e\gamma)$	3.862×10^{-14}	2.730×10^{-14}
$Br(\tau \to \mu\gamma)$	3.860×10^{-8}	2.750×10^{-8}
$Br(\tau \to \mu\mu\mu)$	3.844×10^{-9}	1.147×10^{-9}
$Br(\tau \to \mu ee)$	3.273×10^{-9}	8.788×10^{-10}

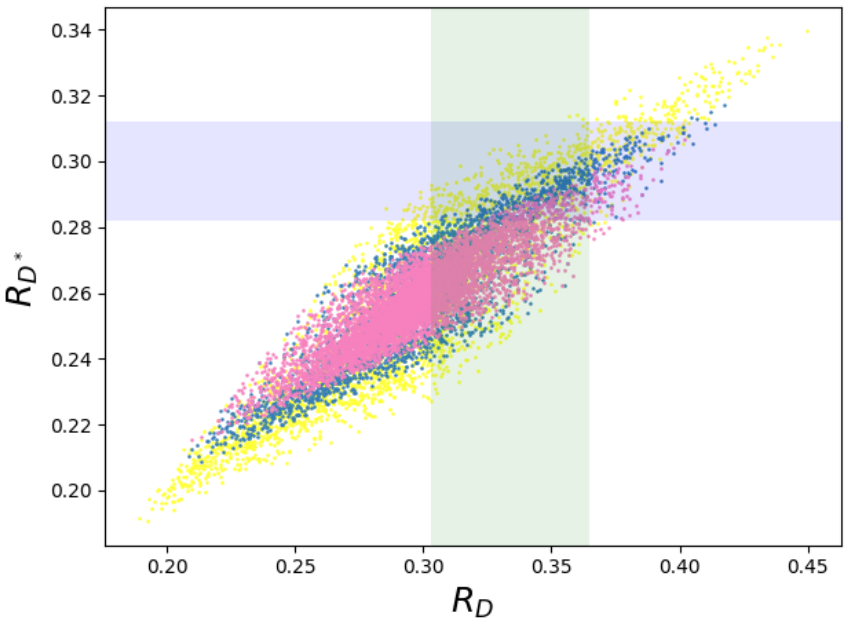
Observables	BM-I	BM-II
$\delta g_R^{\tau\tau}$	1.467×10^{-4}	1.732×10^{-4}
$\delta g_L^{\tau\tau}$	3.955×10^{-5}	6.094×10^{-5}
$Br(Z \to \mu\tau)$	5.363×10^{-15}	1.732×10^{-12}
$Br(B_s \to \mu\mu)$	4.925×10^{-11}	4.954×10^{-11}
$Br(B_s \to \mu\tau)$	3.803×10^{-7}	1.125×10^{-6}
$Br(B_s \to \tau\tau)$	4.604×10^{-5}	5.110×10^{-5}
$Br(B \to K\mu\tau)$	1.063×10^{-6}	3.145×10^{-6}
$R_{K^*}^{\nu\bar{\nu}}$	1.855	1.656
$Br(B_c \to \tau\nu)\%$	1.767	1.703
$\Delta m_{B_s}^{NP+SM}/\Delta m_{B_s}^{\rm SM}$	1.12	1.13
$Br(\tau \to \phi\mu)$	2.047×10^{-8}	1.380×10^{-8}
$Br(\tau \to \eta\mu)$	4.397×10^{-8}	3.123×10^{-9}

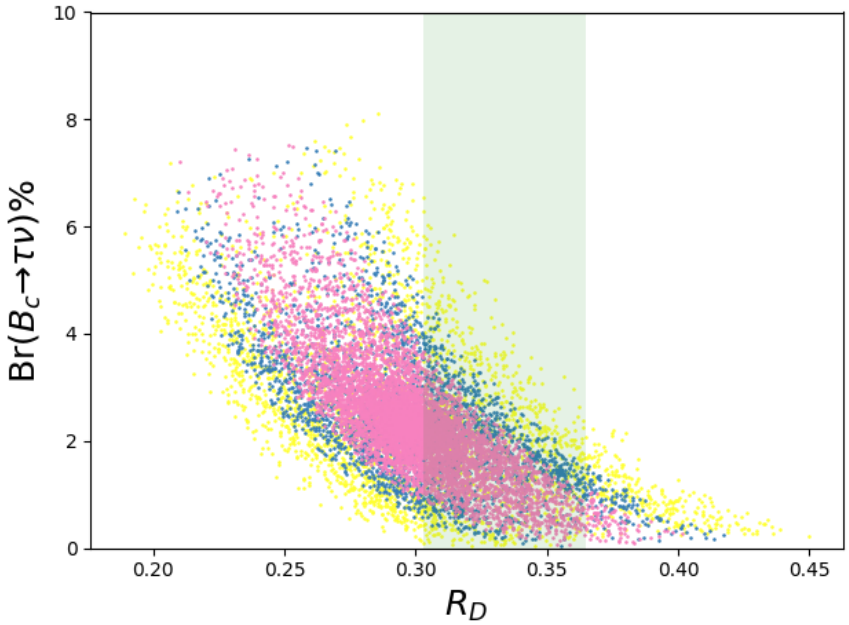
Model-II





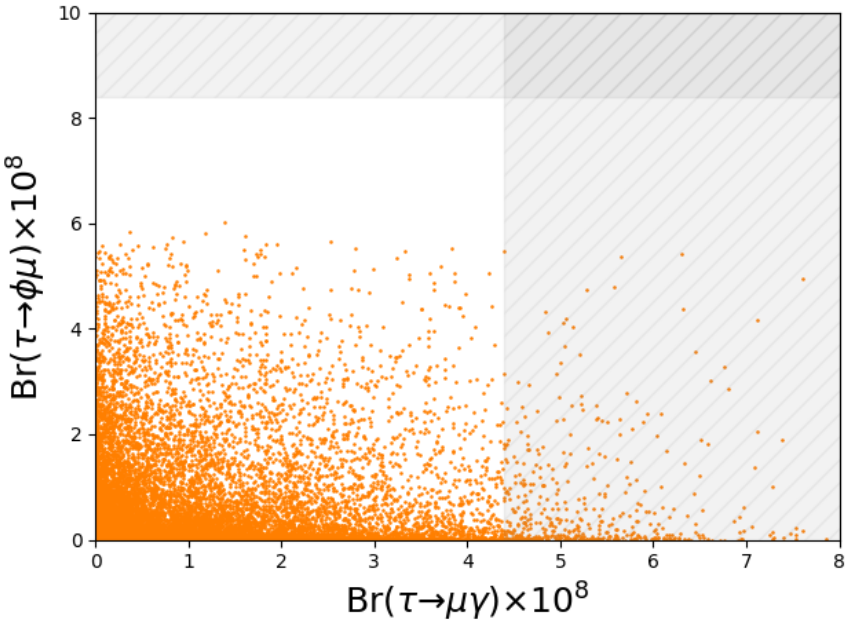
Model-II

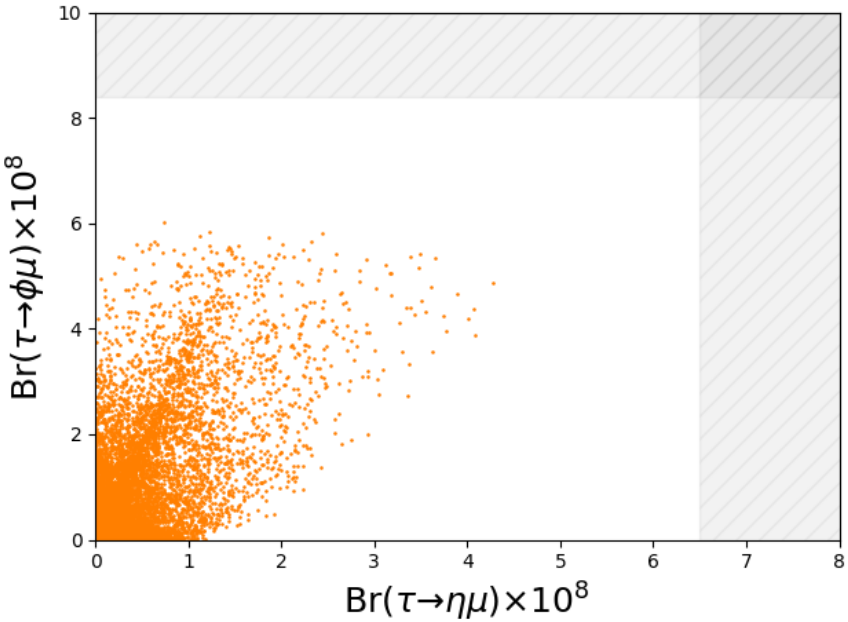




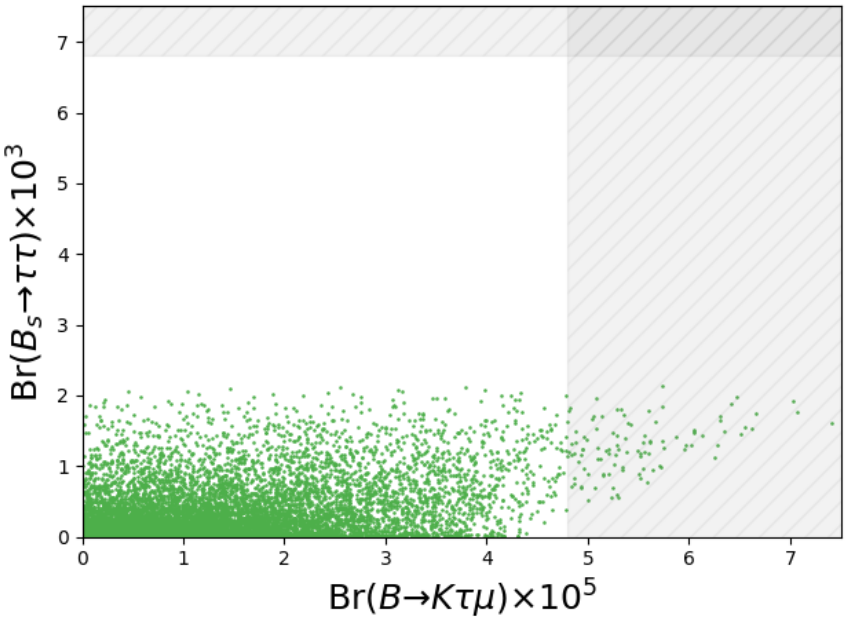
$$B_s^0 - \overline{B_s^0} \sim 10\%, 20\%, 50\%$$

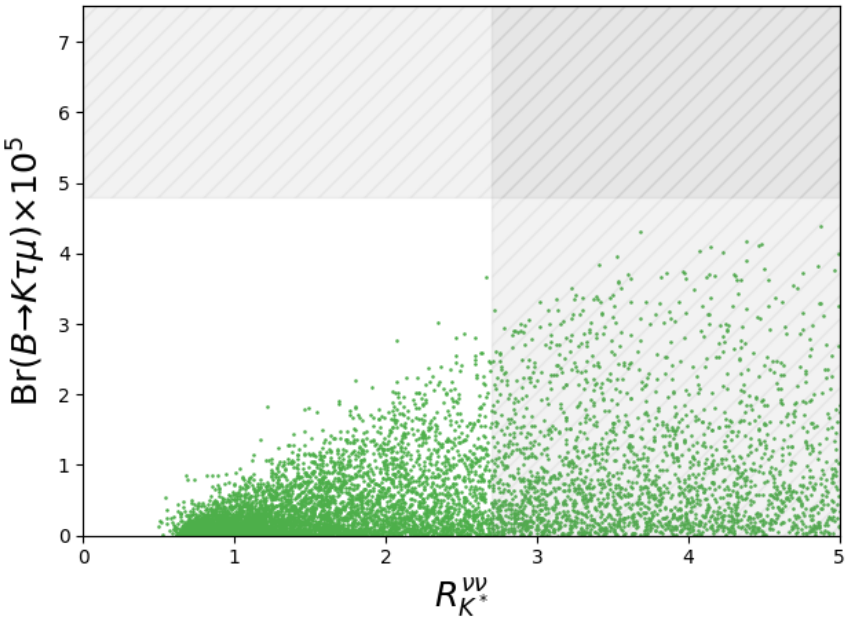
Model-II


Br(τ→φμ)×10^8
Br(τ→μγ)×10^8
0
1
2
3
4
5
6
7
8
10


Br(τ→φμ)×10^8
Br(τ→ημ)×10^8
0
1
2
3
4
5
6
7
8
10

Model-II





Summary

- Minimal extensions

✓ $(g-2)_\mu$

✓ R_D, R_{D^*}

✓ R_K, R_{K^*}

✓ $\mathcal{M}_\nu \neq 0$

- Predictions

✓ TeV scale Leptoquarks (LHC)

✓ Charged lepton flavor violation (upcoming experiments)

✓ Promising Tau and Meson decays (Belle-II, LHCb)