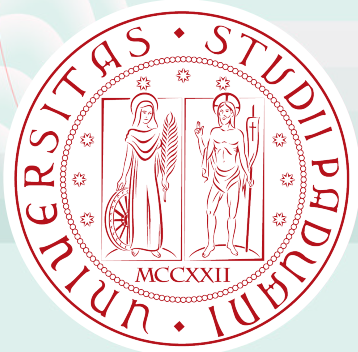


# Charting the Higgs self-coupling boundaries

**Ennio Salvioni**

University & INFN,  
Padua



OSU, 11/17/2022

Based on 2209.00666, 2202.01228, 2110.06941  
with Gauthier Durieux and Matthew McCullough

# Introduction: Higgs couplings

In the Standard Model, the Higgs self-coupling is predicted in terms of other input parameters. At tree level,

$$\mathcal{L}_{\text{SM}} \supset -m_h^2 \sqrt{\frac{G_F}{2\sqrt{2}}} h^3$$

The same applies to the other, single-Higgs couplings, such as for instance  $hZZ$ :

$$\mathcal{L}_{\text{SM}} \supset m_Z^2 \sqrt{\sqrt{2}G_F} h Z_\mu Z^\mu$$

Measuring these interactions and testing whether they agree with the SM or not, is central to LHC physics program

$$\delta_{h^3} \equiv \frac{g_{h^3} - g_{h^3}^{\text{SM}}}{g_{h^3}^{\text{SM}}}$$

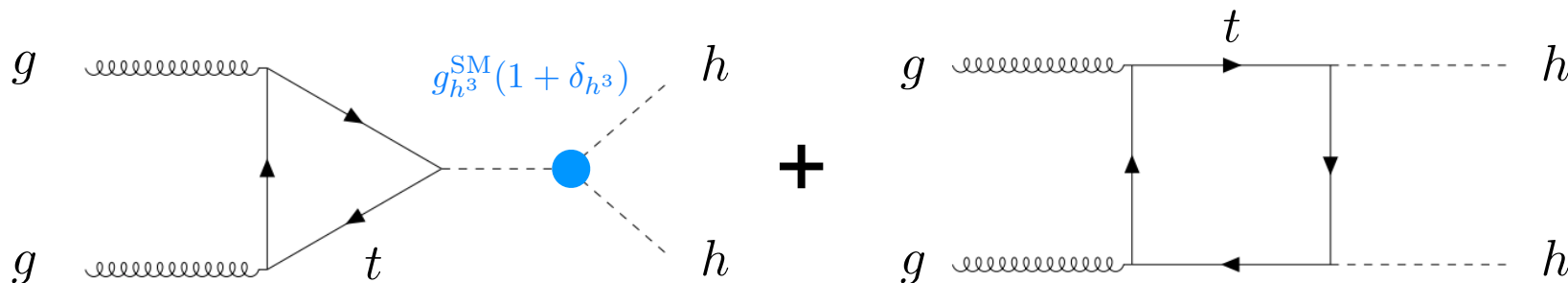
$$\delta_{VV} \equiv \frac{g_{hVV} - g_{hVV}^{\text{SM}}}{g_{hVV}^{\text{SM}}}$$

Are these  
different from zero?

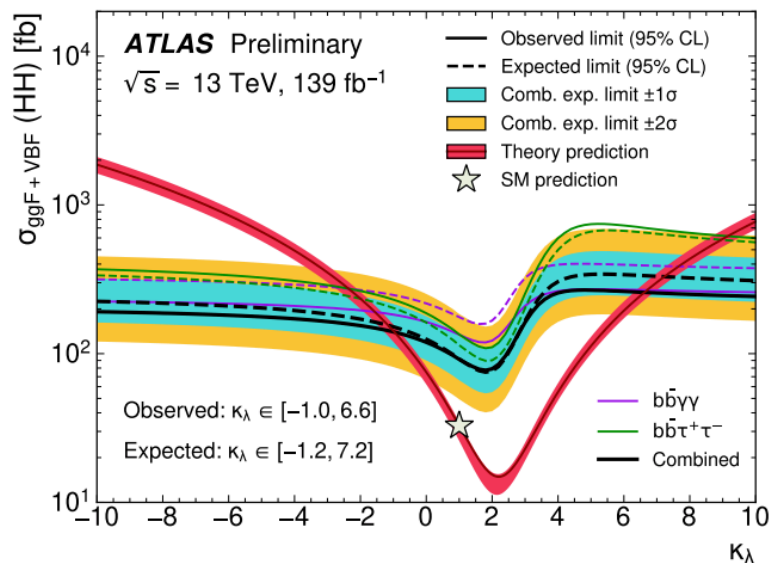
# The Higgs self-coupling

Measuring the Higgs self-coupling at the LHC is huge experimental challenge

Direct access in double Higgs production process:



[ATLAS-CONF-2021-052]



Run 2 measurements constrain ( $2\sigma$ )

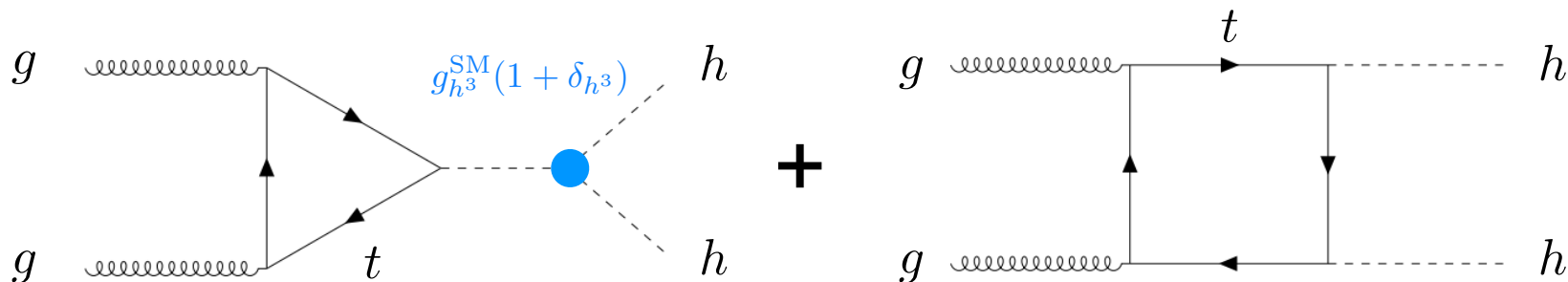
$$-2.0 < \delta_{h^3} < 5.6$$

Very large deviations from SM are still allowed

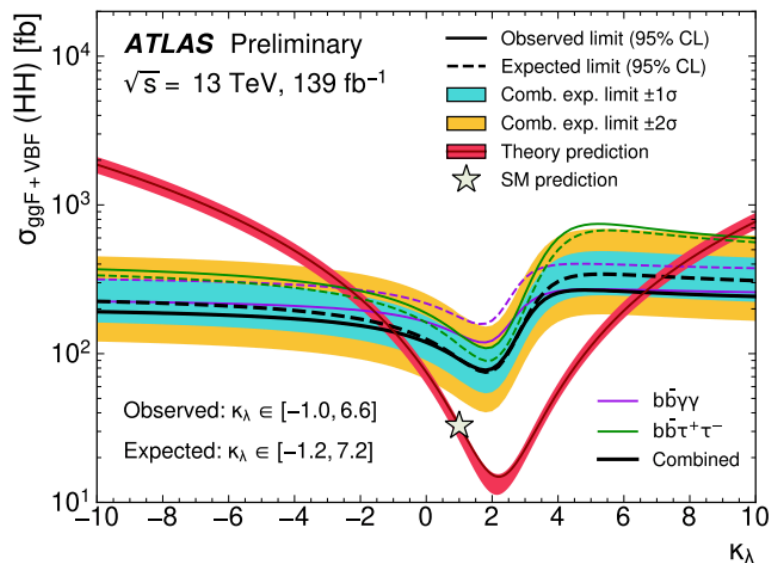
# The Higgs self-coupling

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[ATLAS-CONF-2021-052]



At High-Luminosity LHC, expect 100% precision

(rule out  $\delta_h^3 = 0$  at  $2\sigma$ )

[de Blas et al. 1905.03764]



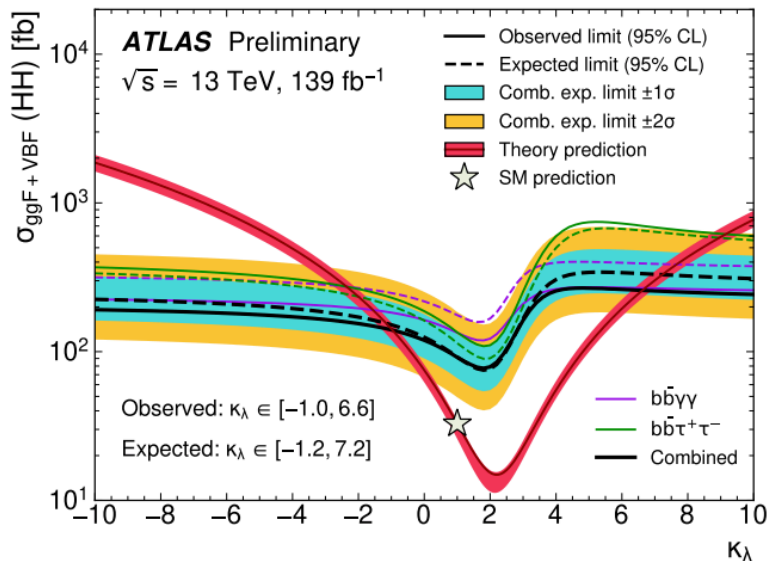
# Single-Higgs couplings

Compare to measurements  
of single-Higgs couplings:

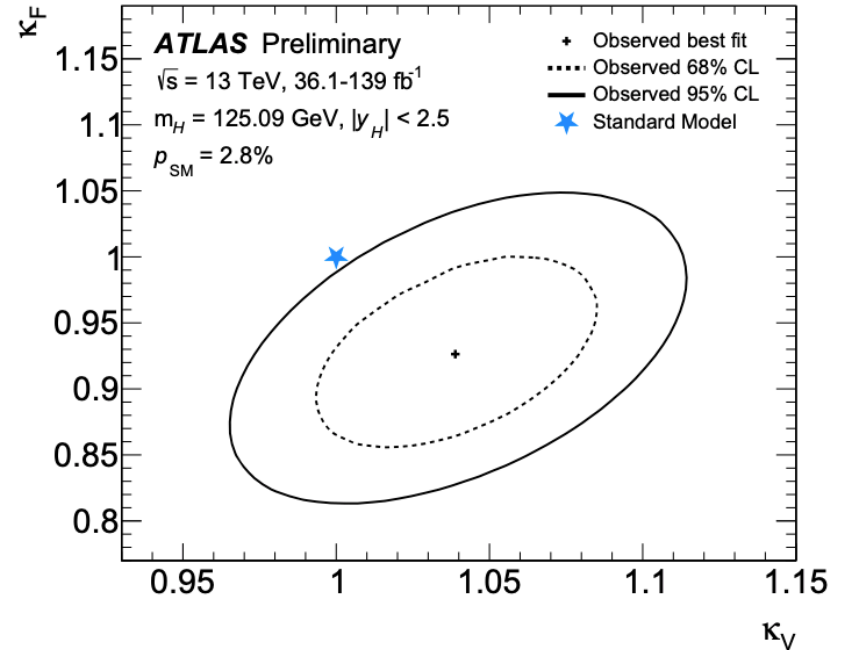
$$|\delta_{VV}| \lesssim 0.07$$

Already now at  $\sim 10\%$  level

[ATLAS-CONF-2021-052]



[ATLAS-CONF-2021-053]



At High-Luminosity LHC, expect 100% precision

(rule out  $\delta_{h^3} = 0$  at  $2\sigma$ )

[de Blas et al. 1905.03764]

# Theory question

Given that measuring the self-coupling is experimentally challenging,  
and that we have not seen new physics so far:

How large can deviations in the Higgs self-coupling be,  
if other (Higgs and electroweak) measurements are compatible with the SM?

To address this question, we ask how large

$$\left| \frac{\delta_{h^3}}{\delta_{VV}} \right|$$

can be in generic ultraviolet completions of the SM

[Durieux, McCullough, Salvioni 2209.00666]

generic = no fine tuning for the purpose of getting a specific value of  $h^3$

generic  $\neq$  canonical model

## Part 1

**An upper bound on  $\delta_{h^3}/\delta_{VV}$**

# An upper bound on $\delta_{h^3}/\delta_{VV}$

Assume new physics is heavy enough to be described by SM effective field theory

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_6 + \mathcal{L}_8 + \dots$$

In terms of higher-dimension operators, a large  $\delta_{h^3}/\delta_{VV}$  corresponds to generating

$$\mathcal{O}_6 = -\frac{c_6}{M^2}|H|^6 \quad \gg \quad \mathcal{O}_H = \frac{c_H}{M^2}(\partial_\mu |H|^2)^2, \quad \mathcal{O}_R = \frac{c_R}{M^2}|H|^2|D_\mu|^2, \quad \mathcal{O}_T = \frac{c_T}{M^2}|H^\dagger D_\mu H|^2$$

$|c_6| \gg |c_{H,R,T}|$ 
single-Higgs couplings
electroweak  $T$  parameter

Note peculiar property of  $c_6$  : only SMEFT coefficient with dimension of (coupling)<sup>4</sup>

$\hbar$ counting	$\int DH e^{iS[H]/\hbar} \rightarrow S \sim \hbar$	$\longrightarrow$	field trilinear coupling loop factor	$H \sim \hbar^{1/2}$ $g_* \sim \hbar^{-1/2}$ $\sim \frac{\hbar}{(4\pi)^2}$
------------------	--	-------------------	--	--

# An upper bound on $\delta_{h^3}/\delta_{VV}$

Imagine a UV completion with mass scale  $M$  and single coupling parameter

$$\kappa = g_*^4 \sim \hbar^{-2}$$

Then  $\hbar$  counting enforces:

$$c_6 \sim \kappa$$



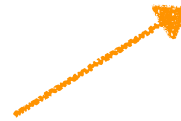
Higgs self-coupling  
at tree level dim-6

$$c_{H,R,T} \sim \frac{\kappa}{(4\pi)^2}$$



single-Higgs couplings and  $T$   
at 1 loop dim-6, or tree level dim-8

$$c_{H_8,R_8,T_8} \sim \kappa$$



$$\mathcal{O}_{H_8,R_8,T_8} \equiv \frac{|H|^2}{M^2} \mathcal{O}_{H,R,T}$$

$$\mathcal{O}_6 = -\frac{c_6}{M^2} |H|^6 \quad \gg \quad \mathcal{O}_H = \frac{c_H}{M^2} (\partial_\mu |H|^2)^2, \quad \mathcal{O}_R = \frac{c_R}{M^2} |H|^2 |D_\mu|^2, \quad \mathcal{O}_T = \frac{c_T}{M^2} |H^\dagger D_\mu H|^2$$

# An upper bound on $\delta_{h^3}/\delta_{VV}$

$$c_6 \sim \kappa \qquad c_{H,R,T} \sim \frac{\kappa}{(4\pi)^2} \qquad c_{H_8,R_8,T_8} \sim \kappa$$

Higgs self-coupling  
at tree level dim-6

single-Higgs couplings and  $T$   
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$$\delta_{h^3} \sim \frac{\kappa v^4}{M^2 m_h^2}$$



$$\delta_{VV}, \hat{T} \sim \frac{\kappa v^2}{M^2} \max \left[ \frac{1}{16\pi^2}, \frac{v^2}{M^2} \right]$$

$$\left| \frac{\delta_{h^3}}{\delta_{VV}} \right| \lesssim \min \left[ \left( \frac{4\pi v}{m_h} \right)^2, \frac{M^2}{m_h^2} \right]$$

reaches  $\approx 600$

for  $M > 4\pi v \approx 3 \text{ TeV}$

Upper bound in generic UV completions

# Implications for phenomenology

$$\left| \frac{\delta_{h^3}}{\delta_{VV}} \right| \lesssim \min \left[ \left( \frac{4\pi v}{m_h} \right)^2, \frac{M^2}{m_h^2} \right] \quad \begin{array}{l} \text{reaches } \approx 600 \\ \text{for } M > 4\pi v \approx 3 \text{ TeV} \end{array}$$

## Some examples

- Now:  $|\delta_{VV}| \lesssim 0.07 \rightarrow |\delta_{h^3}| \lesssim 40$ , compare to double Higgs exp  $-2 < \delta_{h^3} < 5.6$   
HL-LHC:  $|\delta_{VV}| \lesssim 2.6\% \rightarrow |\delta_{h^3}| \lesssim 15$ , compare to exp  $|\delta_{h^3}| \lesssim 100\%$   
→ current and future LHC measurements of  $hh$  probe unexplored territory
- FCC-ee:  $|\delta_{ZZ}| \lesssim 0.34\% \rightarrow |\delta_{h^3}| \lesssim 2$ , compare to indirect FCC-ee  $|\delta_{h^3}| < 48\%$   
and direct FCC-hh  $|\delta_{h^3}| \lesssim 10\%$   
→ even if FCC-ee observes SM, opportunities remain for FCC-hh

# Implications for phenomenology

$$\left| \frac{\delta_{h^3}}{\hat{T}} \right| \lesssim \min \left[ \left( \frac{4\pi v}{m_h} \right)^2, \frac{M^2}{m_h^2} \right] \quad \begin{array}{l} \text{reaches } \approx 600 \\ \text{for } M > 4\pi v \approx 3 \text{ TeV} \end{array}$$

Important caveat:  $T$  parameter

If UV does not have custodial symmetry, electroweak precision bounds severely limit size of self-coupling deviations

Example: FCC-ee  $|\hat{T}| \lesssim 10^{-4} \rightarrow |\delta_{h^3}| \lesssim 6\%$

Loop factor is not enough, need [custodially invariant theory](#)



# Concrete realization: custodial weak quadruplet

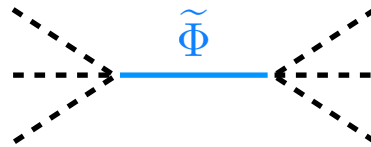
Find a simple extension of the SM, endowed with custodial symmetry,  
saturating  $\delta_{h^3}/\delta_{VV}$  upper bound:

- Weak  $SU(2)_L$  quadruplet couples at renormalizable level to 3 Higgs fields:  $\widetilde{\Phi} \sim 4_{3/2}$

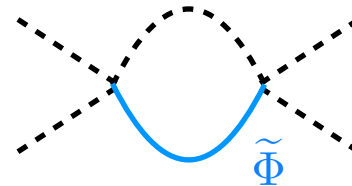
↑  
hypercharge

$$\mathcal{L} \supset -\lambda_{\widetilde{\Phi}} H_i^* H_j^* H_k^* \widetilde{\Phi}_{ijk} + \text{h.c.} + \dots$$

Integrate out:



$$\frac{H^6}{M_{\widetilde{\Phi}}^2}, \quad \frac{D^2 H^6}{M_{\widetilde{\Phi}}^4}$$



$$\frac{1}{(4\pi)^2} \frac{D^2 H^4}{M_{\widetilde{\Phi}}^2}$$

But this includes corrections to  $T$  parameter, @ 1 loop dim-6 or tree level dim-8

- Similarly for the other possible quadruplet,  $\Phi \sim 4_{1/2}$

[de Blas et al. 2014]  
[Henning et al. 2014]  
...

# Concrete realization: custodial weak quadruplet

Combine the two quadruplets into single rep. of custodial symmetry

$$SO(4) \simeq SU(2)_L \times SU(2)_R$$

$$H \quad h_a \sim 4 \quad (\mathbf{2}, \mathbf{2})$$

$$(\Phi, \tilde{\Phi}) \quad \hat{\Phi}^{abc} \sim \mathbf{16} \quad (\mathbf{4}, \mathbf{4})$$

see also [Logan, Rentala 2015]  
[Chala, Krause, Nardini 2018]

$$\mathcal{L} \supset -\lambda \hat{\Phi}^{abc} h_a h_b h_c \quad \rightarrow \quad -\lambda \left( H^* H^* (\epsilon H) \Phi + \frac{1}{\sqrt{3}} H^* H^* H^* \tilde{\Phi} \right) + \text{h.c.}$$

Integrating out, get same operator classes but now  $T = 0$  :

$$\frac{2\lambda^2}{3M^2} |H|^6 \quad \frac{2\lambda^2}{3M^4} (5|H|^4 |D_\mu H|^2 + |H|^2 \partial_\mu |H|^2 \partial^\mu |H|^2)$$

$$\frac{\lambda^2}{12\pi^2 M^2} (8|H|^2 |D_\mu H|^2 + \partial_\mu |H|^2 \partial^\mu |H|^2)$$

Higgs self-coupling  
at tree level dim-6

single-Higgs couplings  
at 1 loop dim-6, or tree level dim-8

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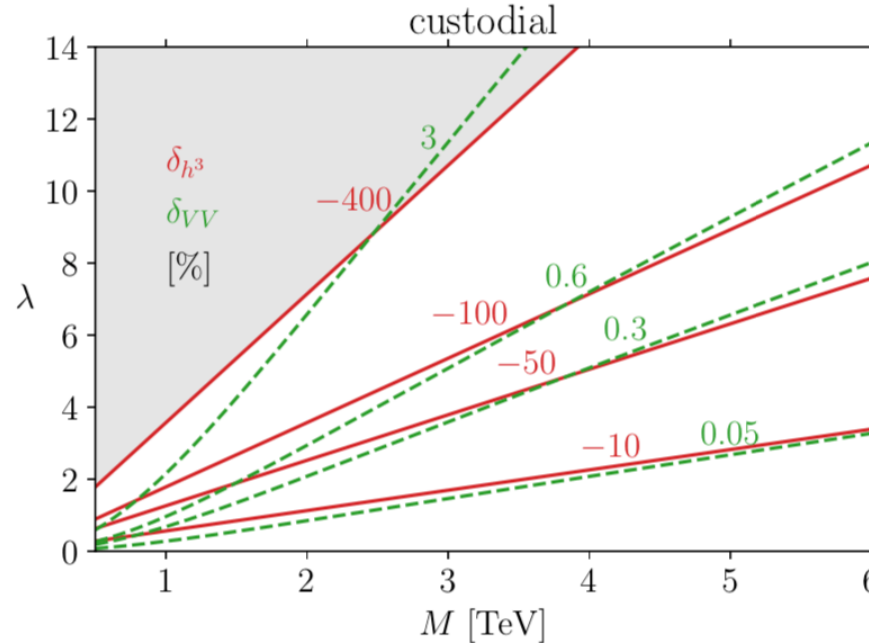
Integrating out, get same operator classes but now  $T = 0$  :

custodial violation happens @ 2 loops dim 6, or 1 loop dim 8,  
negligible



# Custodial weak quadruplet: parameter space

[Durieux, McCullough, Salvioni 2209.00666]



$$\delta_{h^3} \equiv \frac{g_{h^3} - g_{h^3}^{\text{SM}}}{g_{h^3}^{\text{SM}}}$$

$$\delta_{VV} \equiv \frac{g_{hVV} - g_{hVV}^{\text{SM}}}{g_{hVV}^{\text{SM}}}$$

This model indeed **saturates**  
**the upper bound** I discussed:

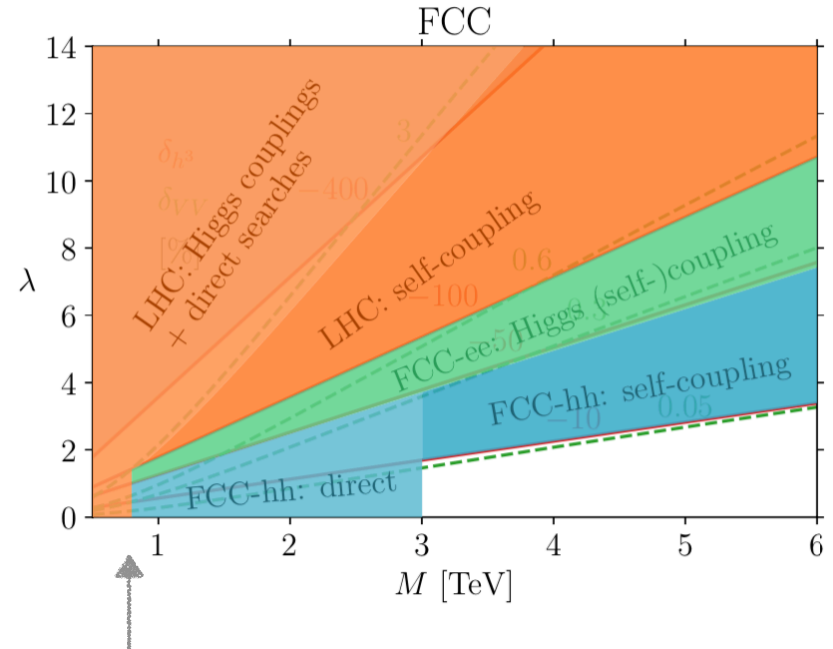
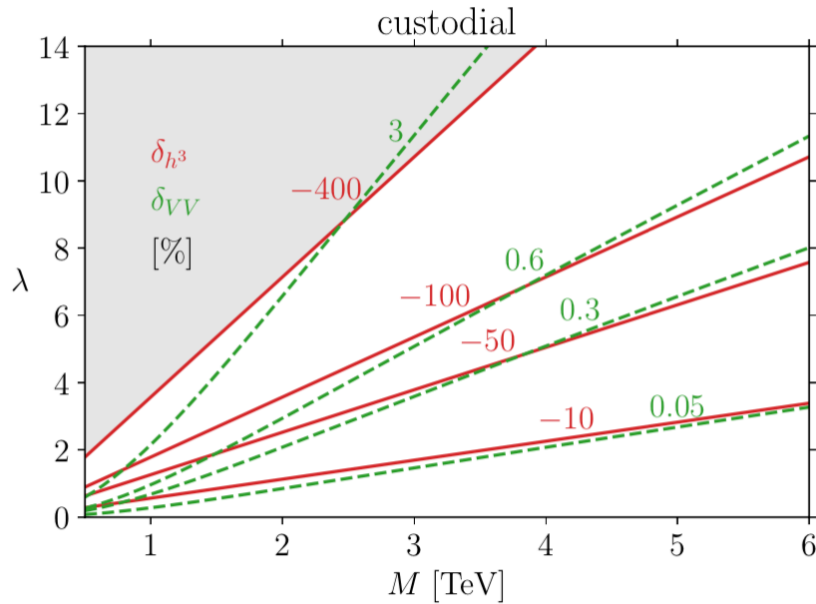
$$-\frac{\delta_{VV}}{\delta_{h^3}} = 3 \left( \frac{m_h}{4\pi v} \right)^2 + \left( \frac{m_h}{M} \right)^2 \approx \frac{1}{200} + \frac{1}{580} \left( \frac{3 \text{ TeV}}{M} \right)^2$$

Recall: key was to have  $\kappa \sim g_*^4$  coupling parameter

Here,  $\lambda \sim g_*^2$  and the  $\lambda \hat{\Phi} h^3$  interaction is invariant under  $\lambda \rightarrow -\lambda$ ,  $\hat{\Phi} \rightarrow -\hat{\Phi}$

→ in EFT where quadruplet is integrated out, **only  $\lambda^2$**  can appear

# Custodial weak quadruplet: prospects



direct HL-LHC reach (pair/single production)

$M \sim 600$  GeV

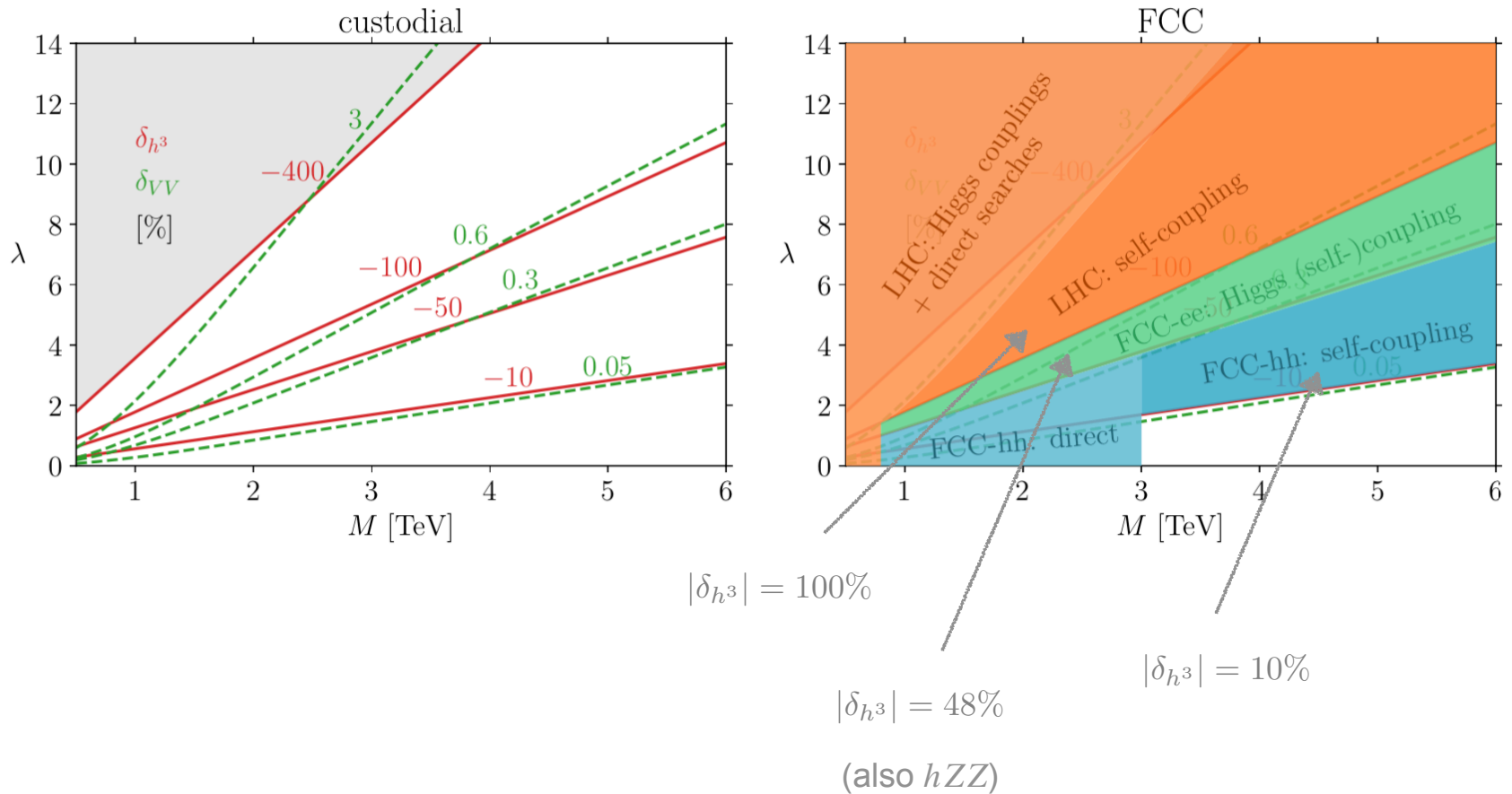
$$\hat{\Phi} = \mathbf{4}_{3/2} + \mathbf{4}_{1/2}$$

$$SU(2)_L \times U(1)_Y$$

By measuring the Higgs self-coupling,

HL-LHC, FCC-ee, FCC-hh will probe wide region of open parameter space

# Custodial weak quadruplet: prospects



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# Vacuum stability

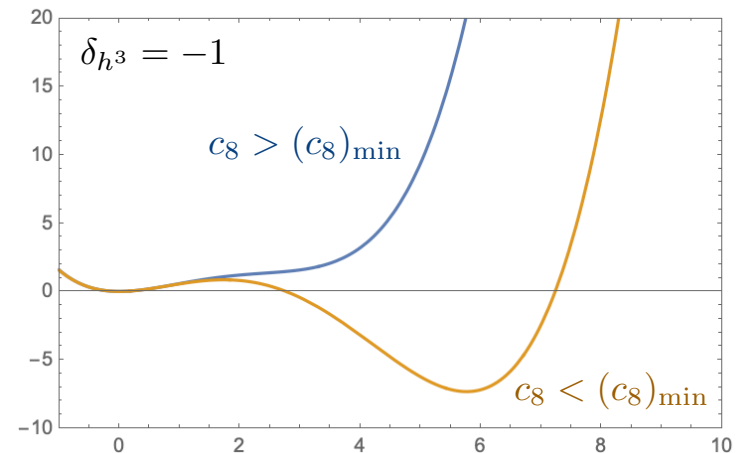
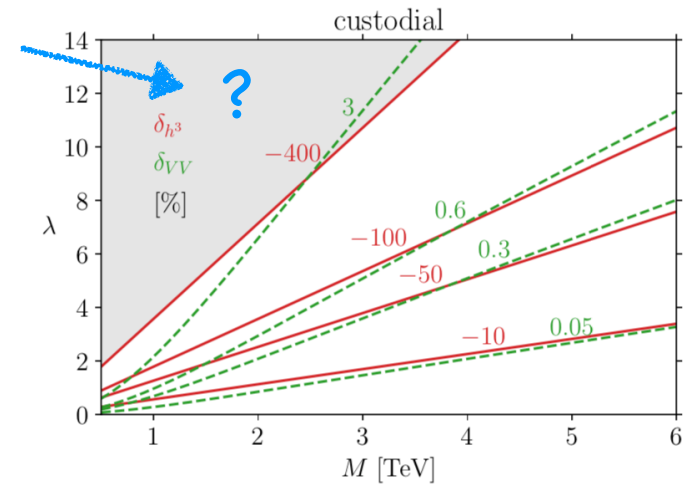
Discussion assumed only the coupling

$$\mathcal{L}_{\text{UV}} \sim \lambda \hat{\Phi} H^3 \quad \longrightarrow \quad \mathcal{L}_{\text{EFT}} \supset \frac{2\lambda^2}{3M^2} |H|^6$$

However, large  $|H|^6$  needs to be accompanied by sizable  $|H|^8$ , to ensure vacuum stability:

$$-\mathcal{L}_{\text{BSM}} = c_6 \frac{|H|^6}{M^2} + c_8 \frac{|H|^8}{M^4}$$

$$c_6 < 0$$



$$\tilde{X} = \frac{2H^\dagger H - v^2}{v^2}$$

# Vacuum stability

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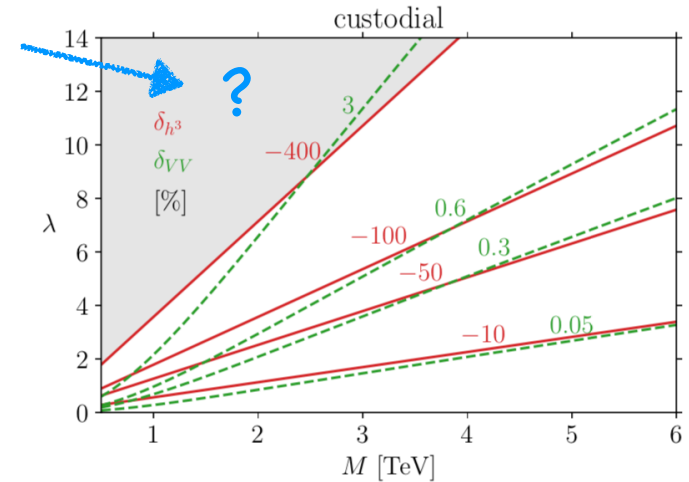
in turn, Higgs self-coupling receives sizable contribution from  $|H|^8$  :

$$\delta_{h^3} = \delta_{h^3}^{(6)} + \delta_{h^3}^{(8)}$$

$$\delta_{h^3}^{(8)} \geq -\delta_{h^3}^{(6)} + 1 - \sqrt{1 - 2\delta_{h^3}^{(6)}}$$

Region shaded in gray is where

$$\left| \frac{\delta_{h^3}^{(8)}}{\delta_{h^3}^{(6)}} \right| \geq 1/2$$



Here EFT based only on  $|H|^6$  becomes unreliable, cannot ignore other UV couplings anymore

For example  $|\hat{\Phi}|^2 |H|^2$ , which leads to  $|H|^8$  at tree level



# Vacuum stability

Discussion assumed only the coupling

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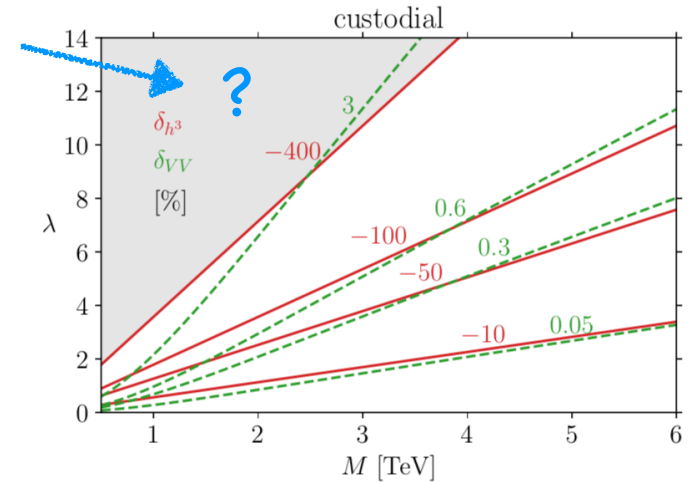
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Expect  $O(1)$  changes in the Higgs couplings and their ratio,  
but qualitatively similar picture



# Summary

How large can deviations in the Higgs self-coupling be,  
if other (Higgs and electroweak) measurements are compatible with SM?

- Starting from simple  $\hbar$  counting observation, derived upper bound in “generic” theories (no specific tuning for  $h^3$ )

$$\left| \frac{\delta_{h^3}}{\delta_{VV}} \right| \lesssim \min \left[ \left( \frac{4\pi v}{m_h} \right)^2, \frac{M^2}{m_h^2} \right]$$

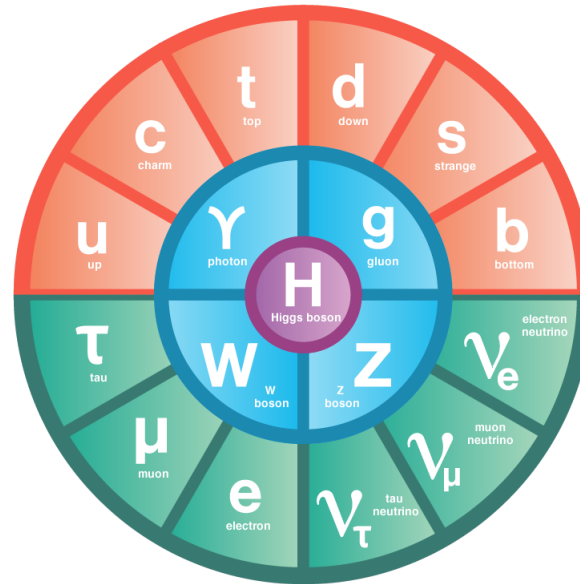
Wrote down concrete model, custodial weak quadruplet, that saturates it

- Shows quantitatively that measuring Higgs self-coupling probes regions of parameters that remain otherwise inaccessible
- On the other hand, quadruplet model is unique example at tree level

## Part 2

**Large  $\delta_{h^3}/\delta_{VV}$  for a pNGB Higgs  
(Gegenbauer's Twin)**

# The Higgs mystery



$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Can we **calculate it** within a more fundamental theory?

# Motivation

Other scalar particles we know: pions

They are composite pseudo Nambu-Goldstone bosons (pNGBs)

$$\Pi \sim (\bar{q}q)$$

$$\Pi \rightarrow \Pi + \theta f$$

Goldstone shift symmetry  
(leading-order transformation  
under broken generators)

$$f \sim 100 \text{ MeV}$$

Old question: could the Higgs field be a (composite) pNGB too?

[Kaplan, Georgi 1984]  
[Kaplan 1992]  
[Agashe, Contino, Pomarol 2004]  
and many, many others

at leading order

$$H \rightarrow H + \theta f$$



$$V(H) = 0$$

$$f \sim \text{TeV}$$

A light Higgs is natural

# The need for $v \ll f$

Leading term of EFT for pNGBs:

$$\mathcal{L} = \frac{f^2}{2} D_\mu \phi^T D^\mu \phi$$

$SO(N+1)/SO(N)$

$$\phi = e^{i\Pi^a T^a / f} \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix} = \begin{pmatrix} \sin \frac{\Pi}{f} \frac{\vec{\Pi}}{\Pi} \\ \cos \frac{\Pi}{f} \end{pmatrix} \quad \Pi \equiv \sqrt{\vec{\Pi}^T \vec{\Pi}}$$

Introducing the SM weak interactions:

$$D_\mu \phi = \partial_\mu \phi - ig W_\mu^a T_L^a \phi - ig' B_\mu T_R^3 \phi$$

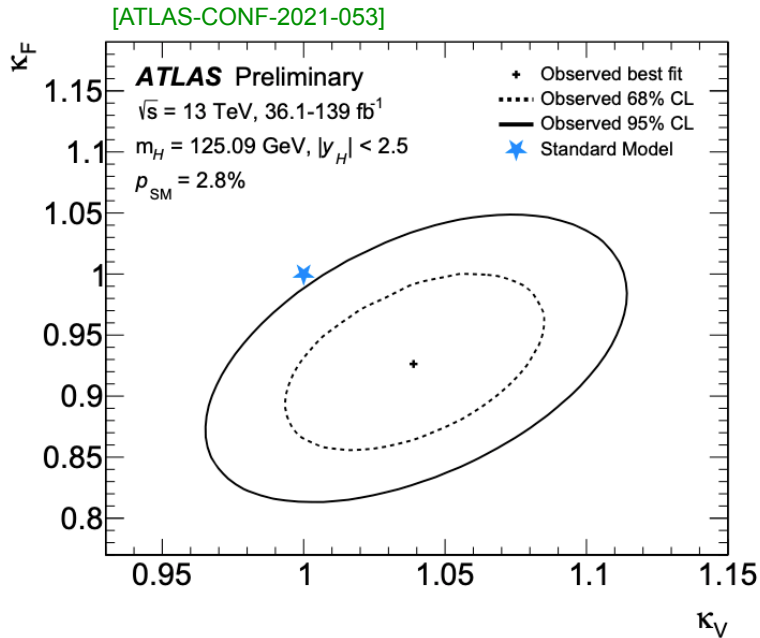


$$\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} = \cos \frac{\langle \Pi \rangle}{f} = \sqrt{1 - \frac{v^2}{f^2}}$$

suppression  
of Higgs couplings  
to other SM particles

$$v \approx 246 \text{ GeV}$$

# The need for $v \ll f$



LHC Run 2:

Higgs couplings agree with SM to  $\sim 10\%$



Need  $v \ll f$  by a factor 3 ~ 4 at least



suppression  
of Higgs couplings  
to other SM particles

$$\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} = \cos \frac{\langle \Pi \rangle}{f} = \sqrt{1 - \frac{v^2}{f^2}}$$

$$v \approx 246 \text{ GeV}$$

# Realizing $v \ll f$

The vast majority of models require **fine-tuning** to achieve it

$$V_{1\text{ loop}} \sim \frac{y_t^2}{16\pi^2} \underbrace{M_T^2}_{\text{top partner mass}} f^2 \left( -\sin^2 \frac{\Pi}{f} + \sin^4 \frac{\Pi}{f} \right) \quad \rightarrow \quad \Delta \sim \frac{v^2}{f^2} \lesssim 10\%$$

**“minimal tuning”** to get  $v \ll f$

[Panico, Redi, Tesi, Wulzer 2012]

Here I present a new class of potentials giving this **naturally**: “Gegenbauer Goldstones”

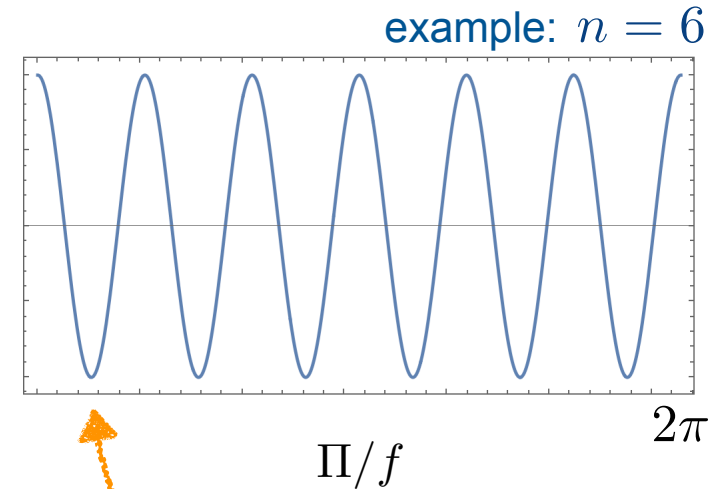
The shape of the Higgs potential is strongly modified compared to SM

 large Higgs self-coupling deviations



# Inspiration: Abelian Goldstone

For a single  $U(1)$  Goldstone,  
we know a simple way to get  $v/f \ll 1$



$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - \lambda (\Phi^* \Phi - f^2)^2$$

$$\frac{\langle \Pi \rangle}{f} = \frac{\pi}{n} \ll 1$$

Make it a pNGB: explicit breaking from operator of charge  $n$

$$\delta V = \frac{\epsilon \lambda}{f^{n-4}} \Phi^n + \text{h.c.}$$



$$\Phi = f e^{i\Pi/f}$$

$$\delta V \sim \epsilon \lambda f^4 \cos \left( \frac{n\Pi}{f} \right)$$

$$\mathcal{Z}_n : \quad \Pi \rightarrow \Pi + \frac{2\pi}{n} f$$

# Non-Abelian Goldstones

Consider  $N$  Goldstone bosons, from spontaneous breaking of global symmetry

$SO(N+1)/SO(N)$  (best studied pattern for pNGB Higgs)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi^T \partial^\mu \Phi - \lambda (\Phi^T \Phi - f^2)^2$$

How to get  $v \ll f$  naturally?

# Non-Abelian Goldstones

Consider  $N$  Goldstone bosons, from spontaneous breaking of global symmetry

$$SO(N+1)/SO(N) \quad (\text{best studied pattern for pNGB Higgs})$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi^T \partial^\mu \Phi - \lambda (\Phi^T \Phi - f^2)^2$$

Explicit breaking to  $SO(N)$  by spurion in  $n$ -index symmetric tensor irrep of  $SO(N+1)$

$$\delta V = \frac{\epsilon \lambda}{f^{n-4}} K_n^{i_1 \dots i_n} \Phi_{i_1} \dots \Phi_{i_n} \quad \text{irrep} \rightarrow \text{traceless}$$

Radiatively stable at  $O(\epsilon)$  and all loop orders, because only operator allowed.

Corrections at  $O(\epsilon^2)$  and higher

[in  $d=2$ : Brézin, Zinn-Justin, Le Guillou 1976]

# Enter Gegenbauer

Parametrize

$$\Phi = f\phi \qquad \phi = e^{i\Pi^a T^a / f} \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix} = \begin{pmatrix} \sin \frac{\Pi}{f} \frac{\vec{\Pi}}{\Pi} \\ \cos \frac{\Pi}{f} \end{pmatrix} \qquad \Pi \equiv \sqrt{\vec{\Pi}^T \vec{\Pi}}$$

$$\delta V = \epsilon \lambda f^4 G_n^{(N-1)/2} (\cos \Pi / f)$$

potential is a  
Gegenbauer polynomial



$$\delta V = \frac{\epsilon \lambda}{f^{n-4}} K_n^{i_1 \dots i_n} \Phi_{i_1} \dots \Phi_{i_n}$$

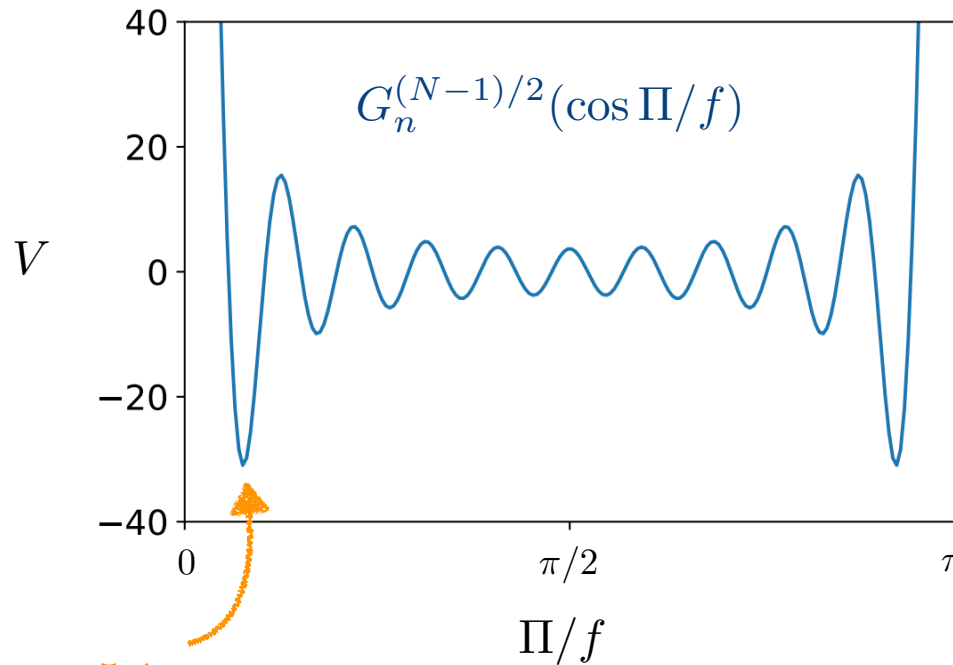
irrep  $\rightarrow$  traceless

Radiatively stable at  $O(\epsilon)$  and all loop orders, because only operator allowed.

Corrections at  $O(\epsilon^2)$  and higher

[in  $d = 2$ : Brézin, Zinn-Justin, Le Guillou 1976]

# The shape of Gegenbauers



$$N = 4$$

$$SO(5)/SO(4)$$

Even  $n$

$$n = 20$$

$$\frac{\langle \Pi \rangle}{f} \approx \frac{j_{N/2,1}}{n + \frac{N-1}{2}} \approx \frac{5.1}{n} \ll 1$$

for large  $n$

Differently from Abelian case,  
not periodic (only approximately)

A radiatively stable way to obtain  
 $\langle \Pi \rangle \ll f$  for non-Abelian Goldstones

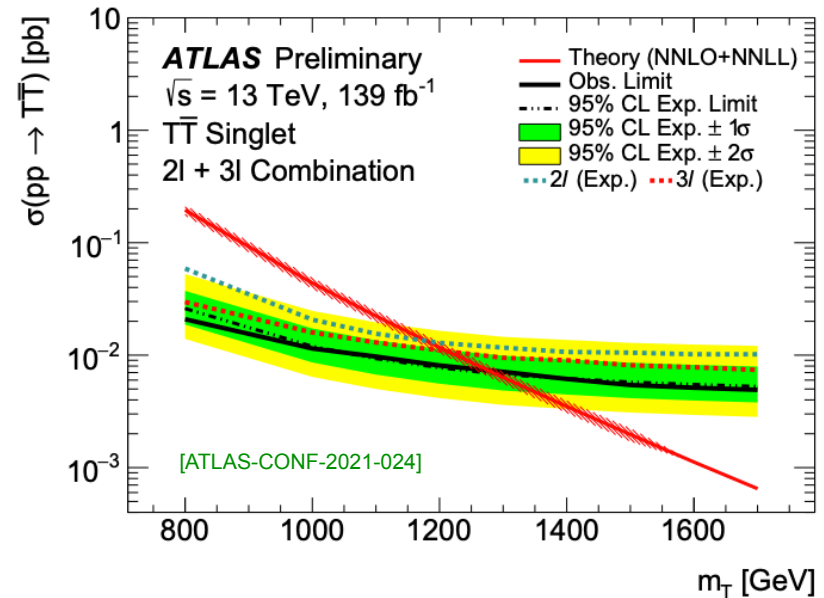
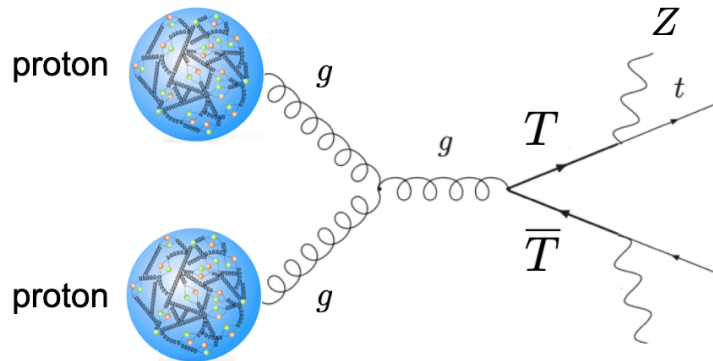
# Gegenbauer Higgs

Gegenbauer potential can realize  $v \ll f$  naturally

[Durieux, McCullough, Salvioni 2110.06941]

But for standard composite pNGB Higgs, some tuning remains:

QCD-charged top partner masses are constrained by LHC data



# Gegenbauer's Twin

Gegenbauer potential can realize  $v \ll f$  naturally

But for standard composite pNGB Higgs, some tuning remains:  
QCD-charged top partner masses are constrained by LHC data

Twin Higgs models can reduce size of top potential: top partners are QCD-neutral

[Chacko, Goh, Harnik 2005]

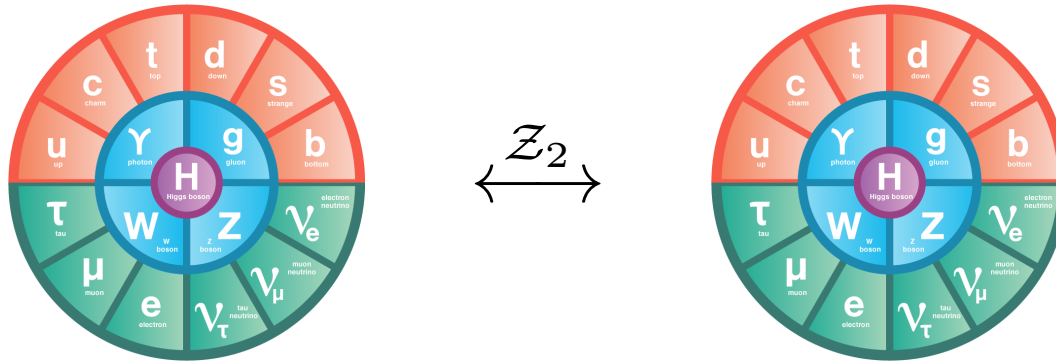
Merge the two: Gegenbauer's Twin



# Twin Higgs

Standard Model

Twin Standard Model



$$\mathcal{L} = y_t Q_A H_A t_A + \hat{y}_t Q_B H_B t_B$$



The top partners are neutral under whole SM (& charged under Twin QCD)  
They can still be really light

The  $\mathbb{Z}_2$  protects Higgs mass from quadratic corrections



# Twin Higgs potential

Quartic terms do not cancel exactly, but resulting potential is not realistic:

$$v = 0 \qquad \text{or} \qquad v = f$$

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2} \left[ \sin^4 \frac{h}{f} \log \frac{a}{\sin^2 h/f} + \cos^4 \frac{h}{f} \log \frac{a}{\cos^2 h/f} \right]$$

# Gegenbauer's Twin

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We introduce a Gegenbauer contribution:

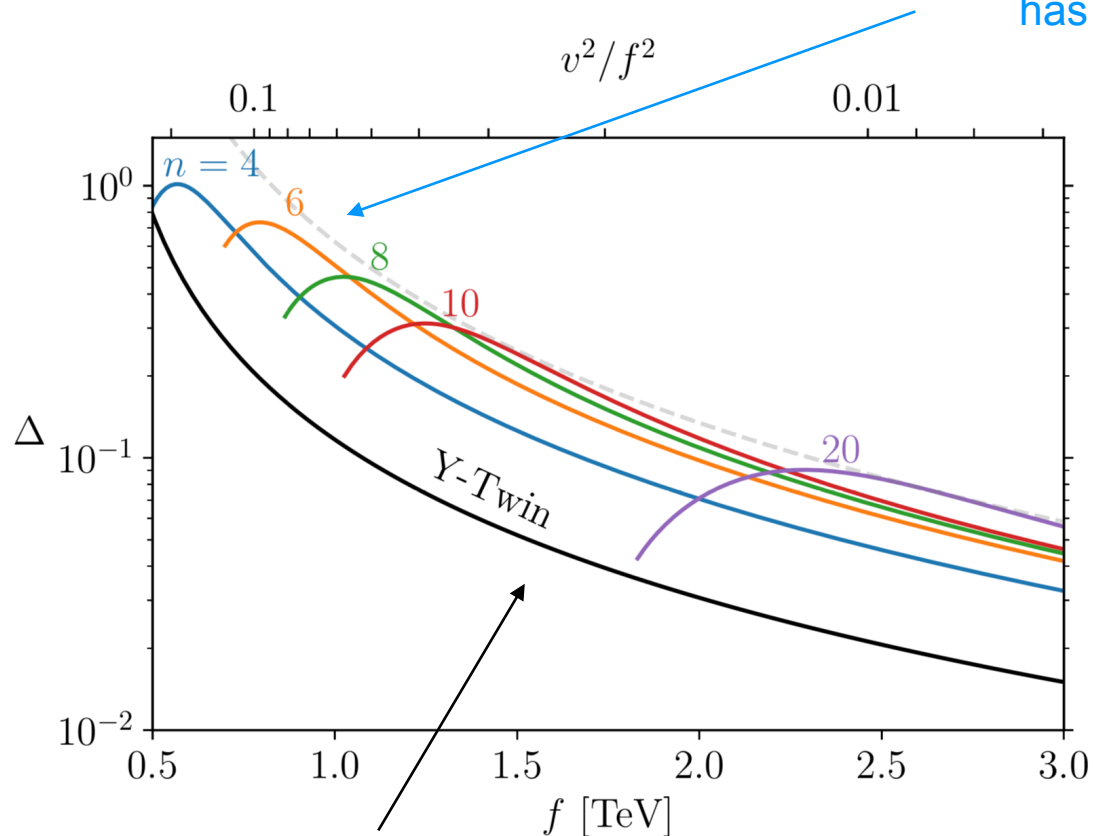
generalize construction to  $SO(8) \rightarrow SO(4) \times SO(4)$  explicit breaking

$$V_G^{(n)} = \epsilon f^4 G_n^{3/2}(\cos 2h/f)$$

# Gegenbauer's Twin

Fine tuning:

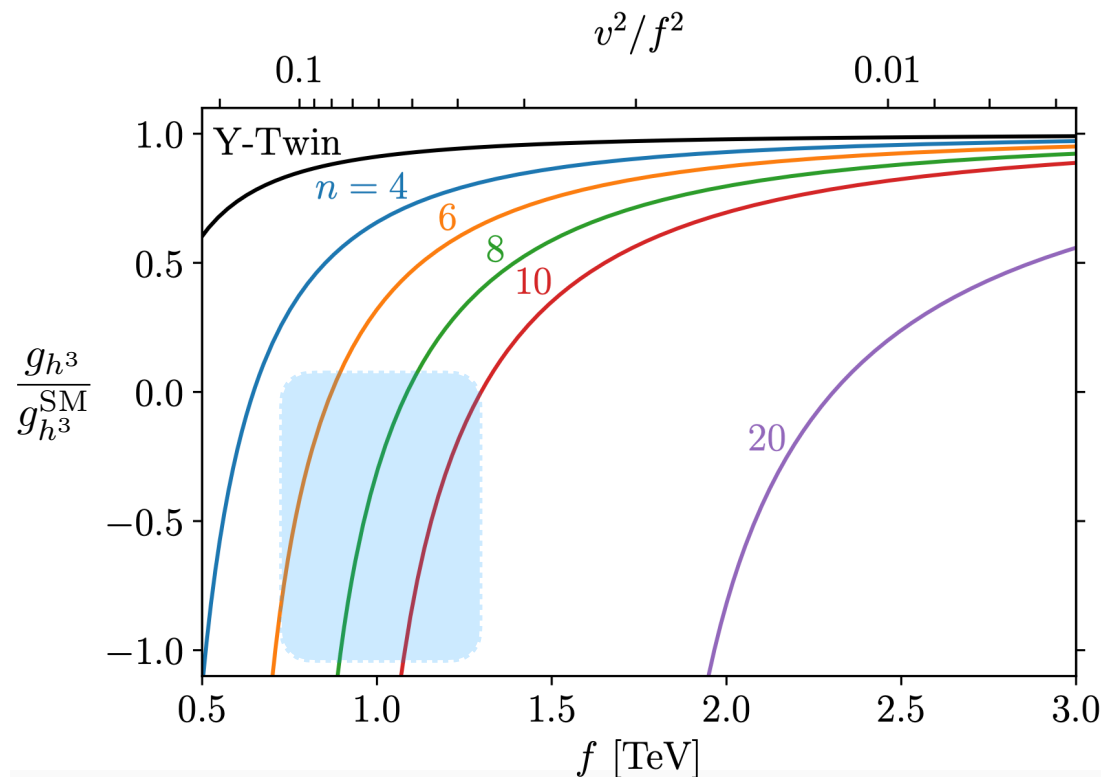
Gegenbauer's Twin with  
 $n = 6$  or  $n = 8$  and  $f \sim 1$  TeV  
has essentially no tuning



standard Twin model (Twin hypercharge not gauged)

# Finally: the Higgs self-coupling

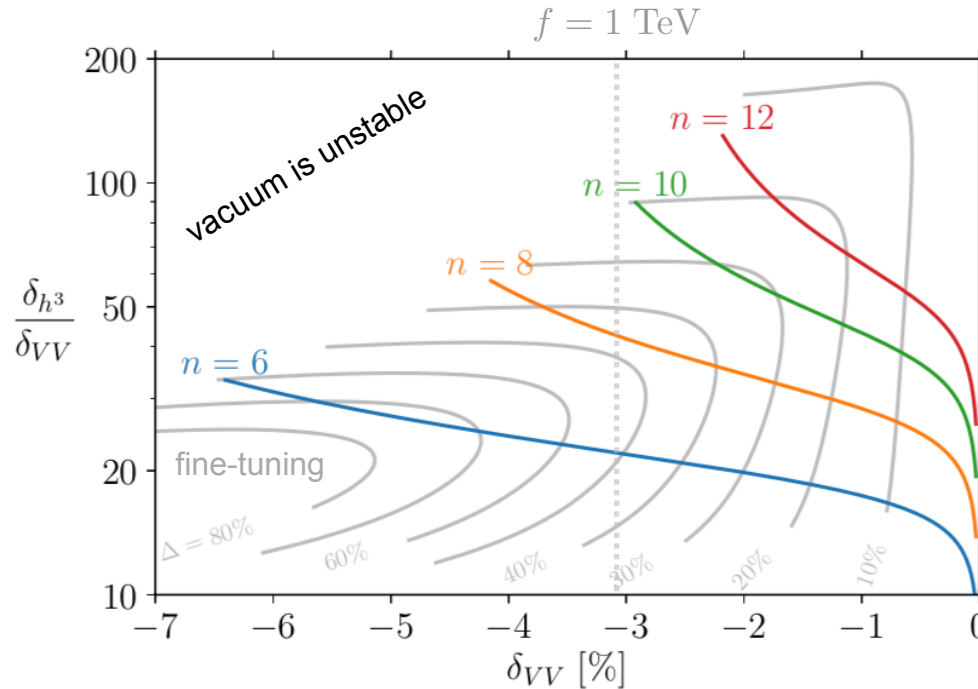
For Gegenbauer's Twin, corrections are parametrically enhanced



“Smoking gun” signal: could even be first deviation observed at LHC

# Back to the $\delta_{h^3}/\delta_{VV}$ ratio

$$\delta_{VV} = \sqrt{1 - \frac{v^2}{f^2}} - 1 \simeq -\frac{v^2}{2f^2}$$



A ratio between 10 and 100 is generic in parameter space of Gegenbauer's Twin

Another class of models where Higgs self-coupling measurements probe new ground

# Conclusions

Gegenbauer's Twin shows that **fully natural electroweak breaking** is still compatible with LHC results

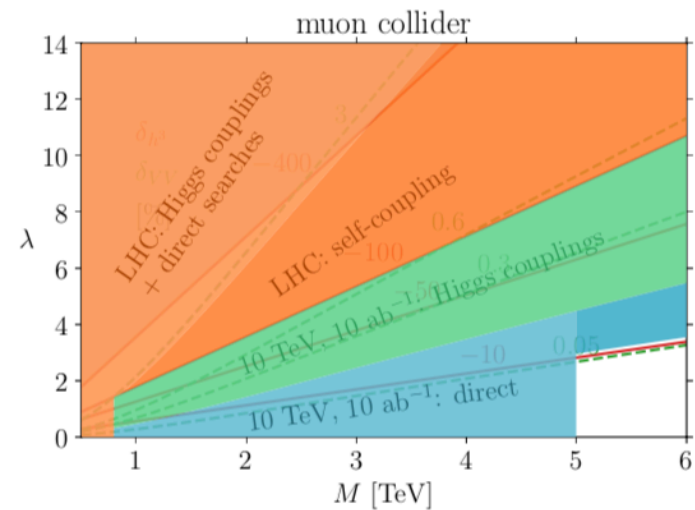
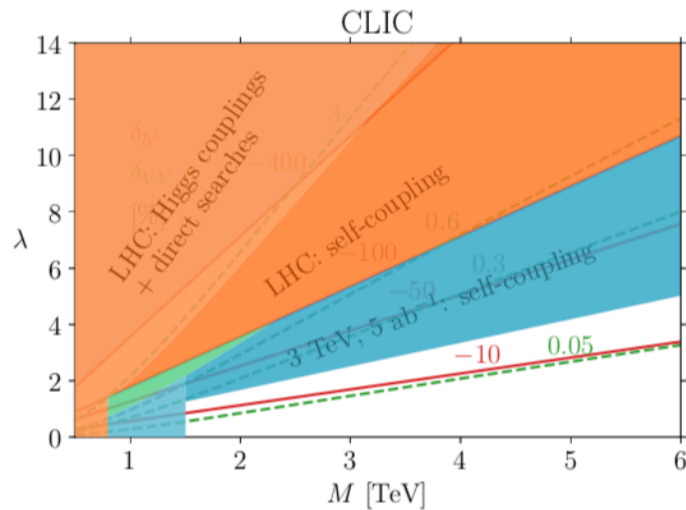
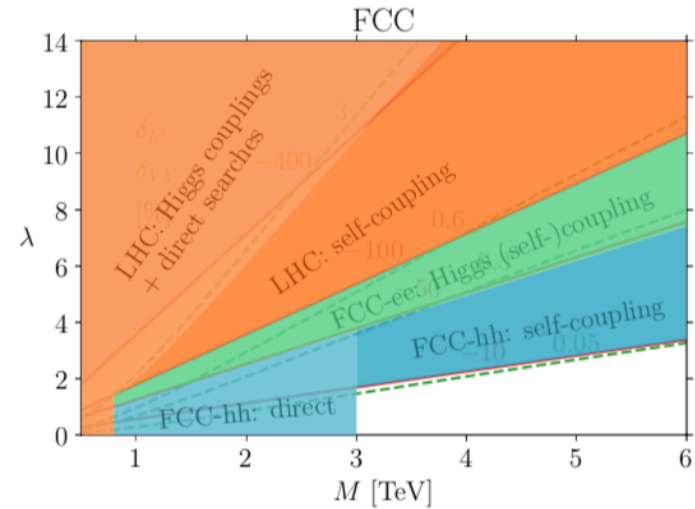
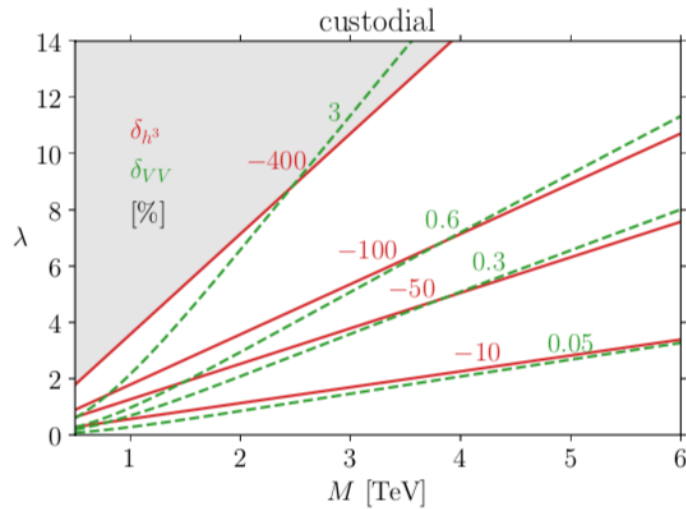
**Large modifications of Higgs self-coupling are generic.**

Ratio  $\delta_{h^3}/\delta_{VV}$  is between 10 and 100, promising for future measurements

Requires to drop often assumed “minimality criteria” about origin of (explicit) symmetry breaking

**Backup slides**

# Custodial quadruplet: HELCs





# Gegenbauer?

Gegenbauer polynomials can be seen as generalization of Legendre polynomials to  $D \neq 3$  spatial dimensions

$$D = 3$$

$$SO(3) \rightarrow SO(2)$$

multipole expansion of axi-symmetric  
function of spacetime coordinates

$$f(\vec{r}) = \sum_{\ell=0}^{\infty} c_{\ell} P_{\ell}(\cos \theta)$$

$(m = 0)$

They appear in many areas of physics, for example in the expansion of conformal blocks in  $\text{CFT}_d$

[Hogervorst, Rychkov 2013]

[Here](#), they arise from explicit breaking of [internal symmetry](#)  $SO(N+1) \rightarrow SO(N)$ , variables are pNGB fields