Discrete Goldstone Bosons

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In Collaboration with V. Enguita Vileta and Belen Gavela (IFT), Pablo Quilez (UCSD), arXiv:2205.09131;

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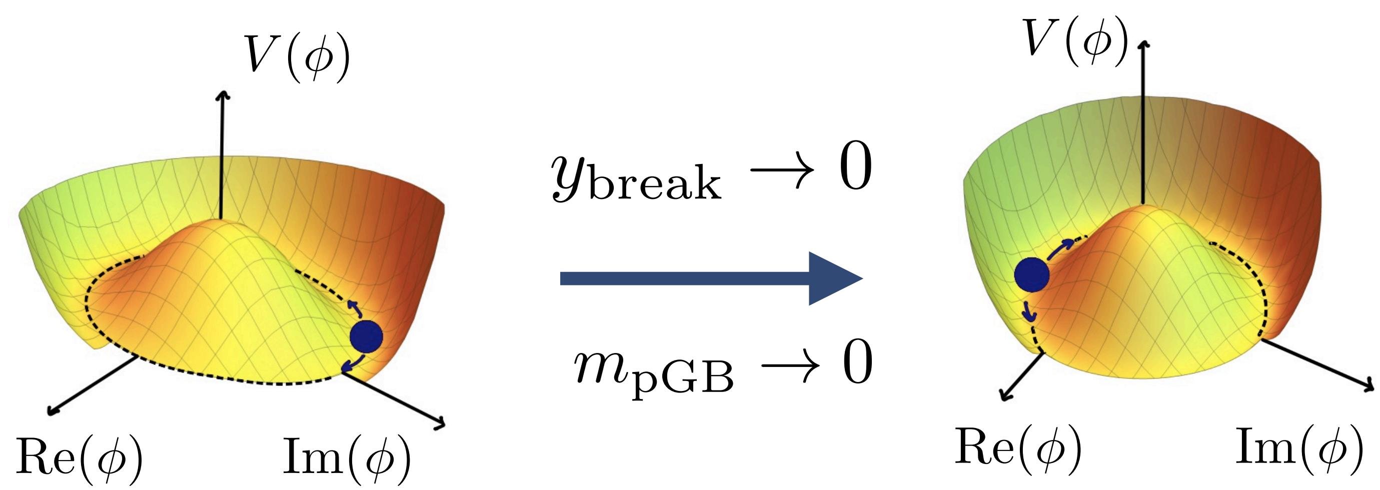
Pseudo-Goldstone Bosons

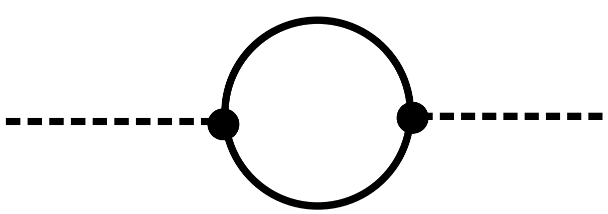
- How can these scales be separated?

$$m_{\text{scalar}}^2 \longleftrightarrow \Lambda_{\text{NP}}^2$$

- Pseudo-Goldstone boson masses are technically natural



- Generically:



$$\Rightarrow \quad m_\pi^2 \sim y_{\text{break}}^2 \Lambda_{\text{NP}}^2$$

- The value of y_{break} is arbitrary

Discrete Symmetries and Pseudo-Goldstone Bosons

- Discrete symmetries may be more fundamental, compatible with gravity
- Additional Z_N symmetries can enhance the mass protection of axionlike pGB fields
- Higher N leads to greater suppression:

Hawking (1975)

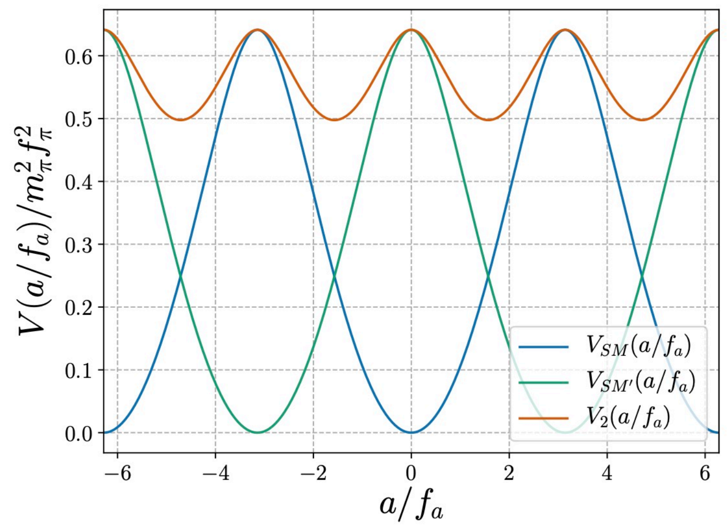
Kallosh, Linde, Linde, Susskind (1995)

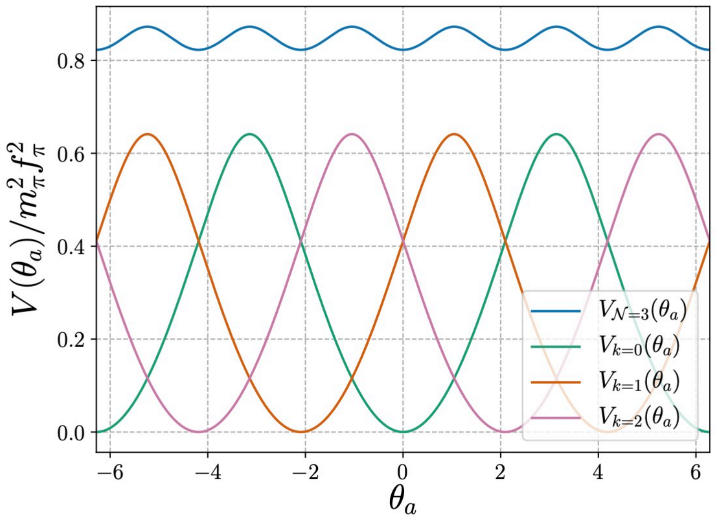
Hook, arXiv:1802.10093

Di Luzio, Gavela, Quilez, Ringwald, arXiv:2102.00012

$$Z_N \xrightarrow{N\to\infty} U(1)$$

$$m_a \to 0$$





Plots lifted from Di Luzio, Gavela, Quilez, Ringwald, arXiv:2102.00012

Non-Abelian Discrete A_4

- A_4 is the simplest non-Abelian discrete group with a three-dimensional irreducible representation
    - ➡ useful in flavor model building, can contain three generations

Ma, Rajasekaran (2001)

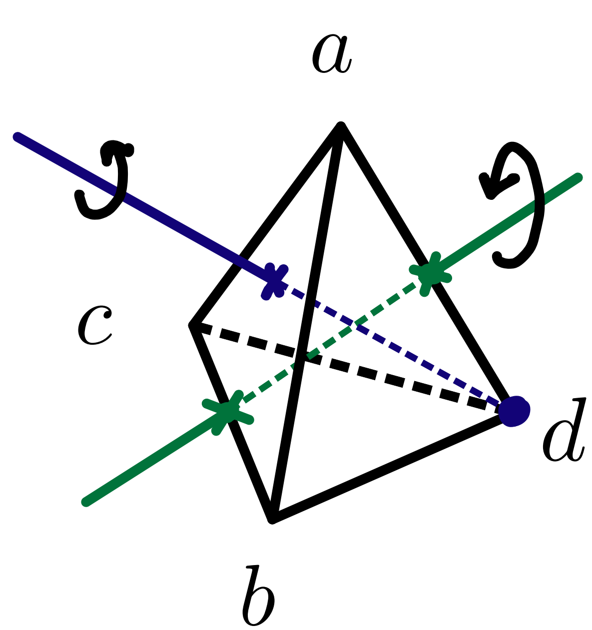
Babu, Ma, Valle (2002)

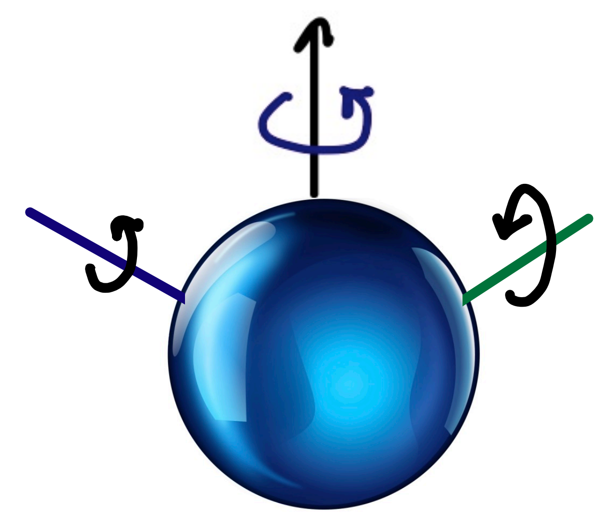
Altarelli, Feruglio (2005)

- A_4 group elements: all even permutations of four objects

$$(a, b, c, d)$$

Even # of swaps





Ishimori, *et al*, arXiv:1003.3552

A_4 is a subgroup of $SO(3)$

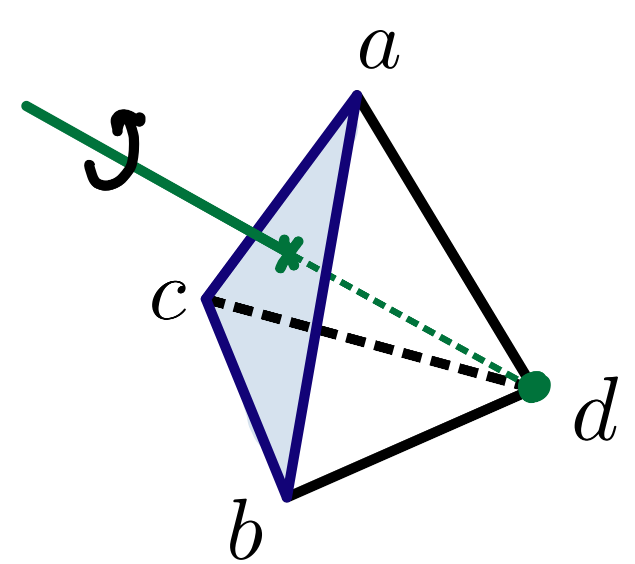
Properties of the A_4 Group

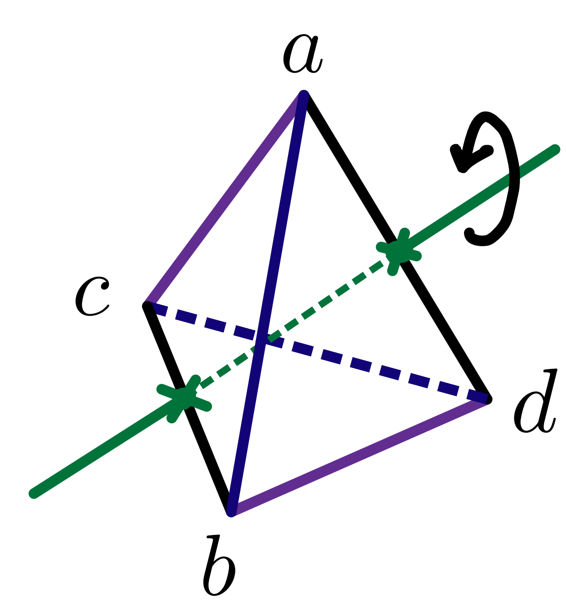
<u>Even permutations of four elements</u>

- t generator: $(a, b, c, d) \to (b, c, a, d)$
  - Eight "face rotations": (t, sts, st, ts) $(t^2, tst, st^2, t^2 s)$
- s generator: $(a, b, c, d) \to (b, a, d, c)$
  - Three "double flips": $(s, t^2 s, tst^2)$

➡ $4!/2 = 12$ total group elements

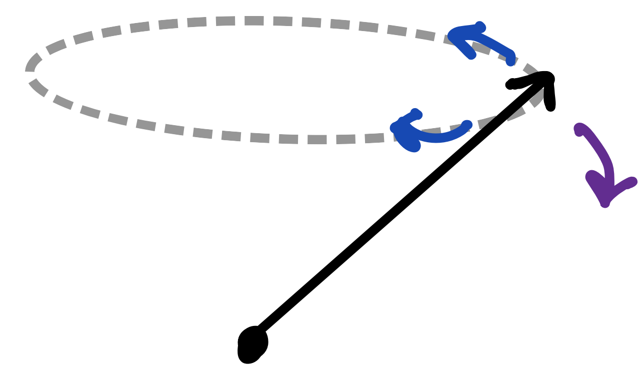
<u>Symmetries of the tetrahedron</u>





Properties of the A_4 Group

- Irreducible representations of A_4:
  - Trivial singlet (invariant)
  - Nontrivial singlets (pick up phases)
  - Triplets



- Of interest to discrete symmetry model builders:

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3'$$

$$A_3 \times B_3 \ni \underbrace{\begin{pmatrix} \{A_y, B_z\} \\ \{A_z, B_x\} \\ \{A_x, B_y\} \end{pmatrix}_3}_{\tilde{\epsilon}_{ijk} A_i B_j} + \underbrace{\begin{pmatrix} [A_y, B_z] \\ [A_z, B_x] \\ [A_x, B_y] \end{pmatrix}_3}_{\epsilon_{ijk} A_i B_j}$$

$$\tilde{\epsilon}_{ijk} = |\epsilon_{ijk}|$$

Nonlinearly Realized Discrete A_4

Das, Hook, arXiv:2006.10767

- The field ϕ has an $SO(3)$ invariant potential in the UV:

$$V(\phi) = -\frac{m^2}{2}\phi^T\phi + \frac{\lambda}{4}\left(\phi^T\phi\right)^2$$

- $SO(3)$ is broken by A_4 invariant Yukawa interactions:

$$\mathcal{L}_{\rm int} = \underbrace{y_a \epsilon^{ijk}\bar{\psi}_i\psi_j\phi_k}_{\text{SO(3) invariant}} + \underbrace{y_s \tilde{\epsilon}^{ijk}\bar{\psi}_i\psi_j\phi_k}_{\text{SO(3) breaking}} \qquad \tilde{\epsilon}_{ijk} = |\epsilon_{ijk}|$$

- Upon SSB:

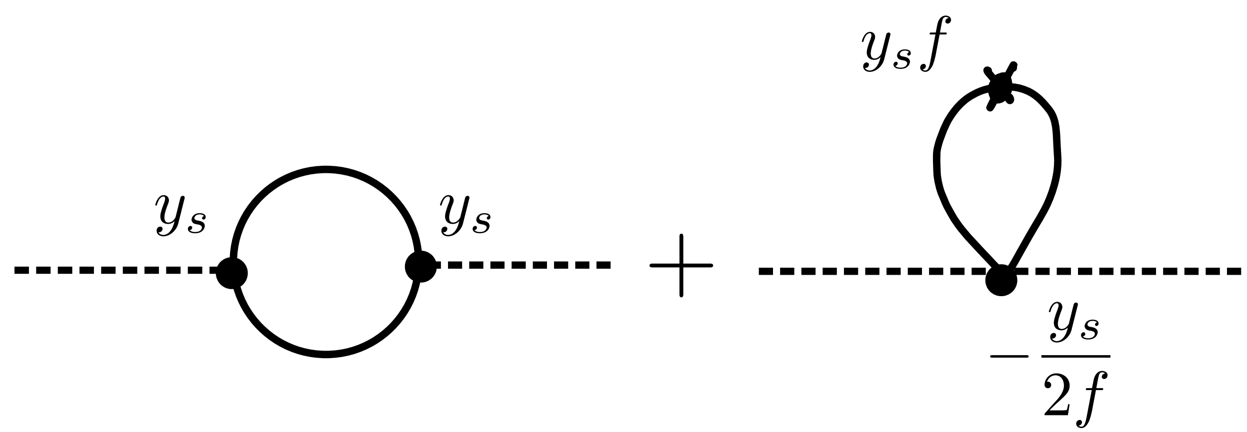
$$SO(3) \to SO(2) \;\Rightarrow\; \quad \phi(\pi_1,\pi_2) = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \exp\left[\frac{1}{f}\begin{pmatrix} 0 & 0 & \pi_1 \\ 0 & 0 & \pi_2 \\ -\pi_1 & -\pi_2 & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

Nonlinearly Realized Discrete A_4

Das, Hook, arXiv:2006.10767

- The quadratically divergent mass contributions stemming from $SO(3)$-breaking terms cancel:

$$\mathcal{L}_{\text{int}} = y_s \pi_1 \left(\bar{\Psi}_2 \Psi_3 + \bar{\Psi}_3 \Psi_2\right) + y_s \pi_2 \left(\bar{\Psi}_3 \Psi_1 + \bar{\Psi}_1 \Psi_3\right) + y_s f \left(1 - \frac{1}{2}\frac{\pi_1^2 + \pi_2^2}{f^2}\right)\left(\bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1\right)$$

$\delta m_\pi^2 \ni$ 

- Similar to Little Higgs models

Arkani-Hamed, Cohen, Gregoire, Wacker (2002)

Arkani-Hamed, Cohen, Katz, Nelson (2002)

➡ How general is this?

General Nonlinear Discrete Symmetries

- Consider a general scalar field Φ in an irreducible m-dimensional real representation of some discrete group D:

$$\Phi \equiv (\phi_1, \phi_2, ..., \phi_m)$$

- Nonlinearity constraint: $\Phi^T\Phi = \phi_1^2 + \phi_2^2 + ... + \phi_m^2 = f^2$

- Reduces the number of dof's by 1, resulting in a set of $m-1$ light spin-0 particles

    - These are the **discrete Goldstone Bosons (dGB)**

    - If D is embedded in a continuous group G, these are a class of pseudo-Goldstone bosons

The EFT for A_4 dGBs

- Consider a scalar field in the triplet of A_4: $\Phi \equiv (\phi_1, \phi_2, \phi_3)$

- The full A_4 invariant potential is a function of the primary invariants:

$$\mathcal{I}_2 = \phi_i \phi_i = \phi_1^2 + \phi_2^2 + \phi_3^2 \longleftarrow \text{SO(3) invariant}$$

$$\left.\begin{aligned} \mathcal{I}_3 &= \prod_{i<j<j} \phi_i \phi_j \phi_k = \phi_1 \phi_2 \phi_3 \\ \mathcal{I}_4 &= \sum_i \phi_i^4 = \phi_1^4 + \phi_2^4 + \phi_3^4 \end{aligned}\right\} \text{SO(3) breaking}$$

- There is also a secondary invariant: one non-polynomial combination of the primaries

$$4\mathcal{I}_6^2 = 2\mathcal{I}_4^3 - 5\mathcal{I}_4^2\mathcal{I}_2^2 + 4\mathcal{I}_4\mathcal{I}_2^4 - 36\mathcal{I}_4\mathcal{I}_3^2\mathcal{I}_2 - \mathcal{I}_2^6 + 20\mathcal{I}_3^2\mathcal{I}_2^3 - 108\mathcal{I}_3^4$$

The EFT for A_4 dGBs

- The most general potential can be written as a *polynomial* of the primary and secondary invariants

$$V_{\rm dGB} = f^2\Lambda^2 \sum_{\substack{a,b,c\\ c=0,1}}^{\infty} \hat{c}_{abc} \left(\frac{\mathcal{I}_3}{f^3}\right)^a \left(\frac{\mathcal{I}_4}{f^4}\right)^b \left(\frac{\mathcal{I}_6}{f^6}\right)^c$$

<u>The primary invariants</u>

$$\mathcal{I}_2 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

$$\mathcal{I}_3 = \phi_1\phi_2\phi_3$$

$$\mathcal{I}_4 = \phi_1^4 + \phi_2^4 + \phi_3^4$$

- We can expand this out in terms of (non-primary) invariants

$$V_{\rm dGB} = f^2\Lambda^2 \left[\hat{c}_3\frac{\mathcal{I}_3}{f^3} + \hat{c}_4\frac{\mathcal{I}_4}{f^4} + \hat{c}_6\frac{\mathcal{I}_6}{f^6} + \hat{c}_7\frac{\mathcal{I}_7}{f^7} + \ldots\right], \qquad \Lambda \leq 4\pi f$$

- Below Λ, the nonlinearity constraint holds: $\mathcal{I}_2 = f^2$

The dGB Potential

- The most general potential:

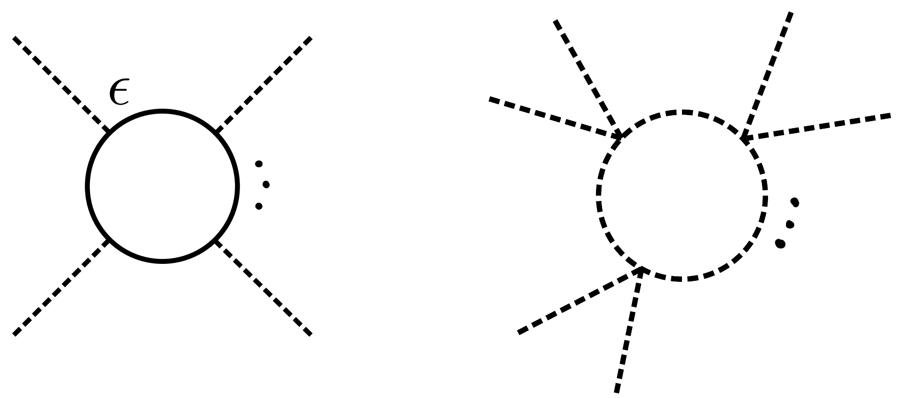
$$V_{\mathrm{dGB}} = f^2\Lambda^2\left[\hat{c}_3\frac{\mathcal{I}_3}{f^3} + \hat{c}_4\frac{\mathcal{I}_4}{f^4} + \hat{c}_6\frac{\mathcal{I}_6}{f^6} + \hat{c}_7\frac{\mathcal{I}_7}{f^7} + \ldots\right]$$

- If the $\hat{c}_n$ are all $\mathcal{O}(1)$, then all terms will contribute equally

- It is easy, however, to arrange for lower order terms to dominate

- If the invariant operators are generated by renormalizable interactions:

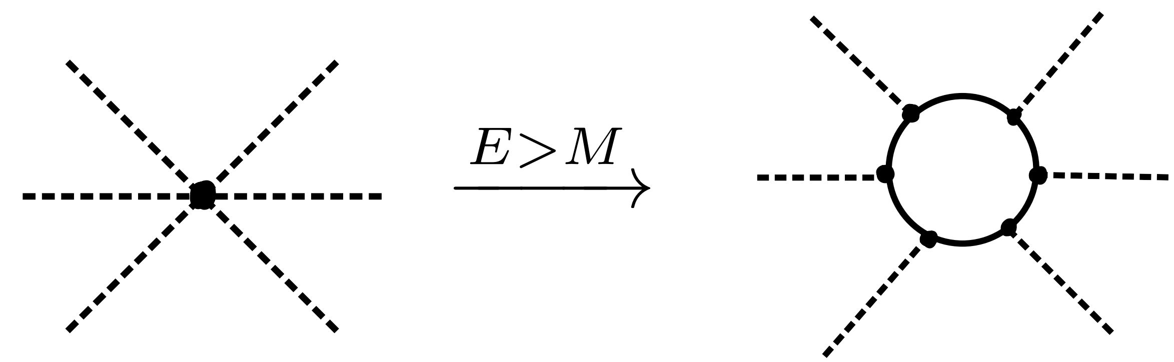
$$\hat{c}_n \sim \epsilon^n$$



- Being a *little* less agnostic about the UV theory can allow us to talk about scales that are not f

Additional $\hat{c}_n$ Suppression

- Consider that higher dimensional operator interactions are mediated by a fermion with mass M



$$\mathcal{L} = \frac{M^4}{16\pi^2} \sum_n \left(y \frac{\Phi}{M} \right)^n = \Lambda^2 f^2 \sum_n y^n \left(\frac{\Lambda}{M} \right)^{n-4} \left(\frac{\Phi}{\Lambda} \right)^n \qquad (\Lambda \lesssim 4\pi f)$$

- Allows us to estimate the sizes of $\hat{c}_n$: $\quad \hat{c}_n \sim y^n \left(\frac{\Lambda}{M} \right)^{n-4}$

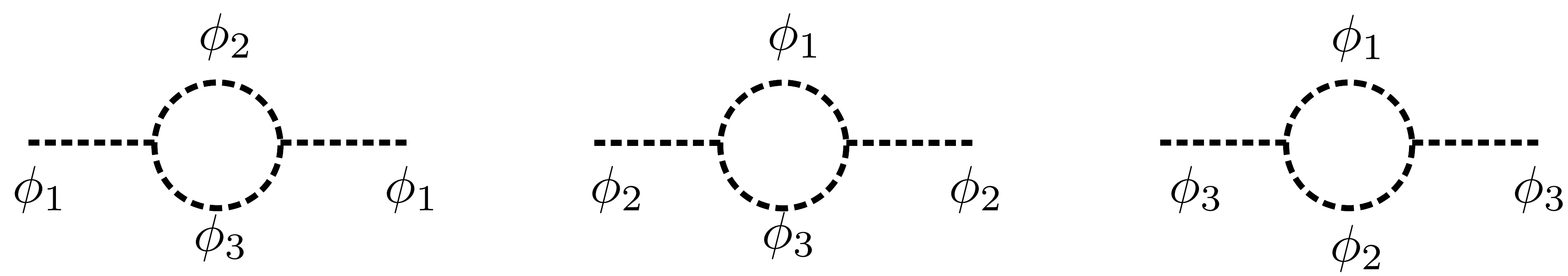
- We can say that the $n > 4$ terms will be subdominant so long as either:
    - $y < 1$
    - $M > \Lambda$

(for $\mathcal{I}_3 > \mathcal{I}_4$, we must assume $y < 1$)

The dGB Potential

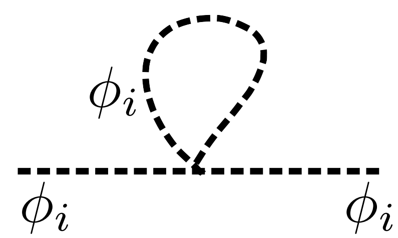
- Invariant interactions generate seemingly destabilizing mass contributions:

$\mathcal{I}_3$:



$\propto \mathcal{I}_2$

$\mathcal{I}_4$:



$\propto \mathcal{I}_2$

- These contributions are rendered harmless by the nonlinearity constraint

$$\Phi^T \Phi = \phi_1^2 + \phi_2^2 + \phi_3^2 = f^2$$

$$\Rightarrow \mathcal{I}_2 = f^2$$ ➡ Non-dynamical

- Nothing proportional to $\mathcal{I}_2$ will generate mass terms for the dGBs (but other invariants will)

The Minima of V_{dGB}

- Next, we want to parameterize the low energy theory after V_{dGB} takes a vev

- $V(\Phi)$ is a function of the primary invariants:

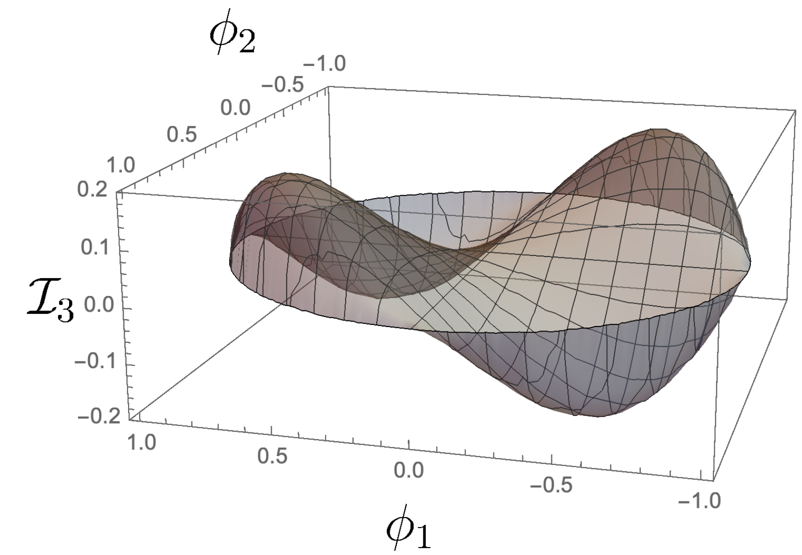
$$V(\Phi) = f(I_3, I_4)$$

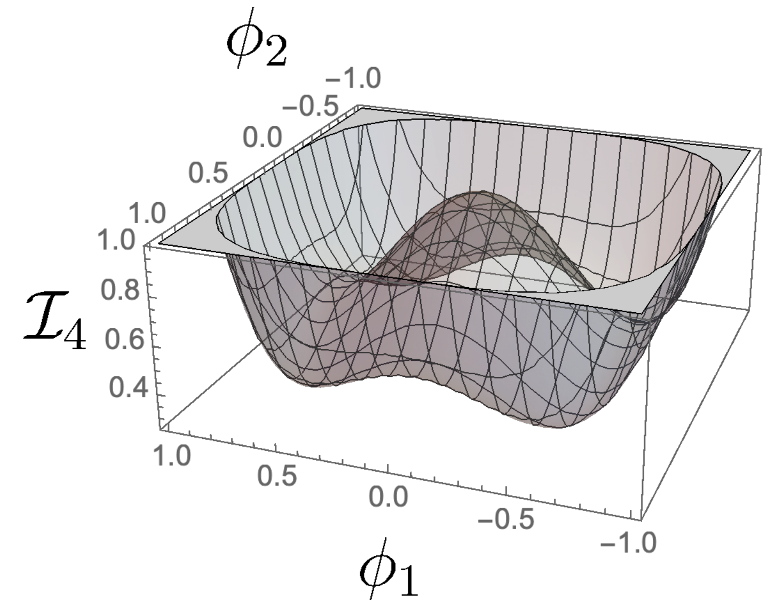
- The critical points of V occur when:

$$\frac{\partial V}{\partial \phi_j} = \sum_i \frac{\partial V}{\partial I_i} \frac{\partial I_i}{\partial \phi_j} = 0$$

Depends on the particular form of the potential

Depends on the structure of the invariants





The Minima of V_{dGB}

- Two different types of extrema: $$\frac{\partial V}{\partial \phi_j} = \sum_i \frac{\partial V}{\partial I_i}\frac{\partial I_i}{\partial \phi_j} = 0$$

1) **Model-dependent extrema:** the position of the minimum depends on a specific combination of parameters in the potential.

   ➡ Example: SM Higgs

2) **Natural extrema:** Extrema of the invariants themselves

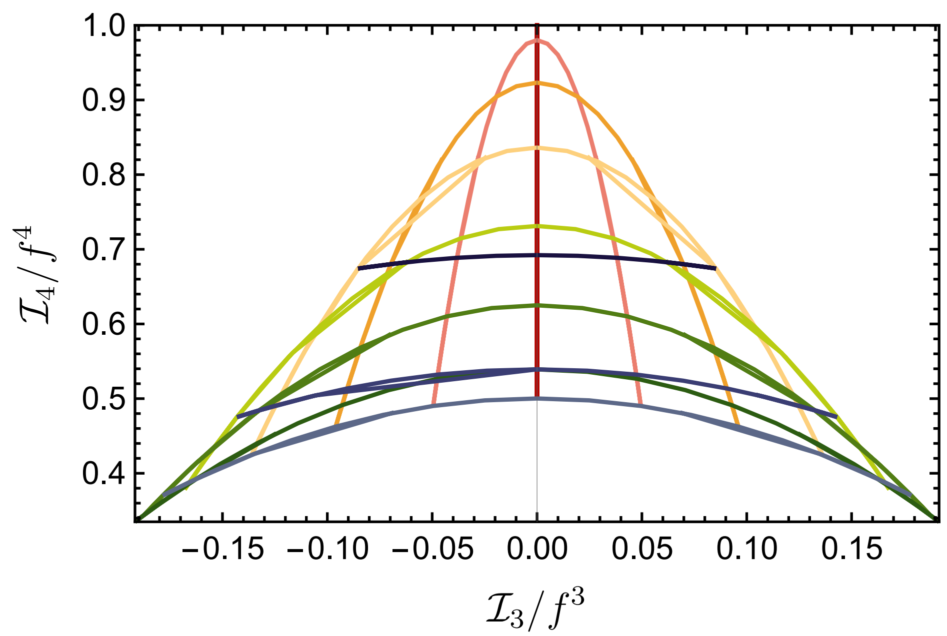
$$\det J = 0 \quad , \qquad J_{ij} = \frac{\partial \mathcal{I}_i}{\partial \phi_j}$$

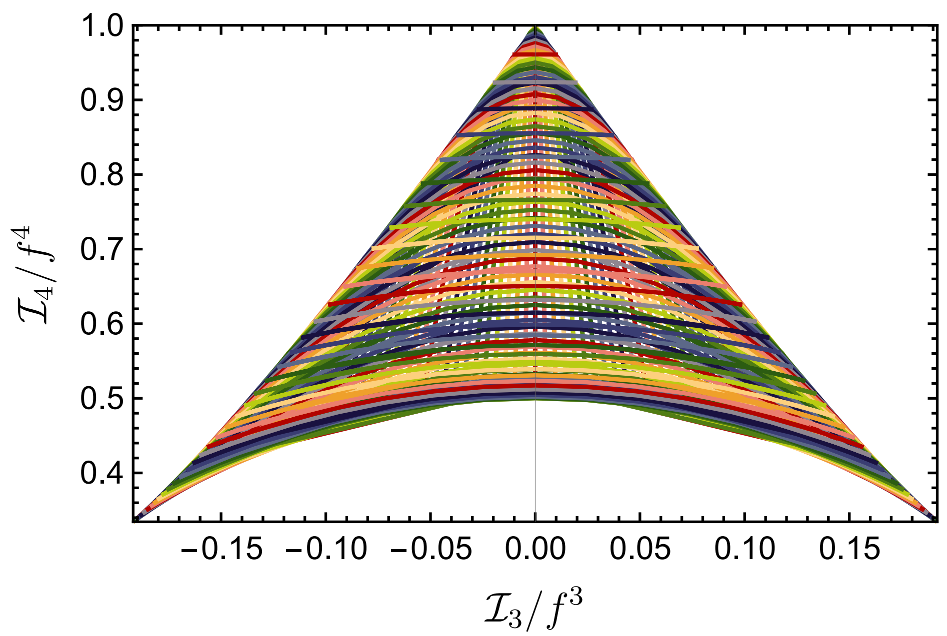
   ➡ Only depend on the discrete symmetry

   ➡ Reduction in rank of the Jacobian indicates preserved symmetry at that minimum

The Natural Minima

- The space spanned by $\mathcal{I}_3$ and $\mathcal{I}_4$ given the nonlinearity constraint is bounded
- At the boundaries, the trajectories must reverse, and $\partial\mathcal{I}_i/\partial\phi_j$ is forced to change signs

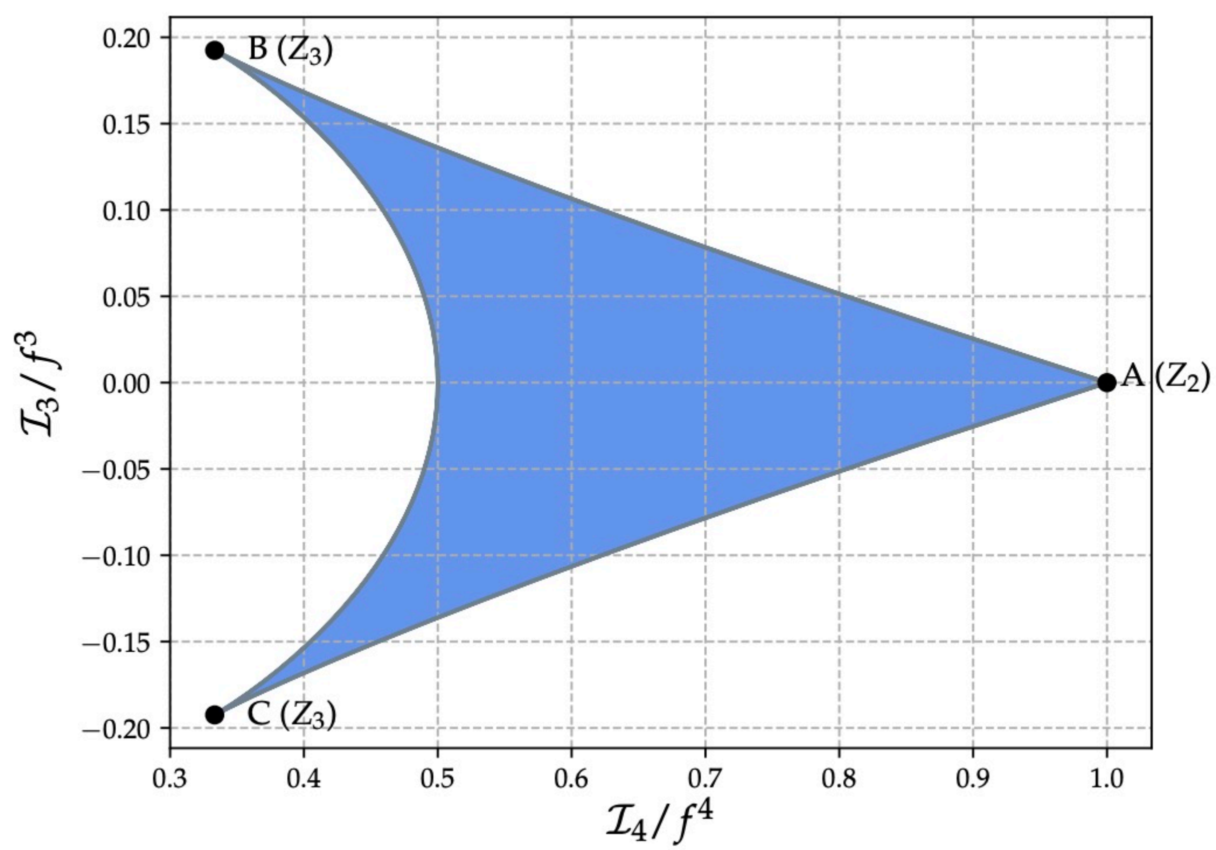




- The natural extrema live on the boundaries. Where the edges meet at points give **maximally natural extrema**

The Natural Minima

Manifold of the A_4 invariants



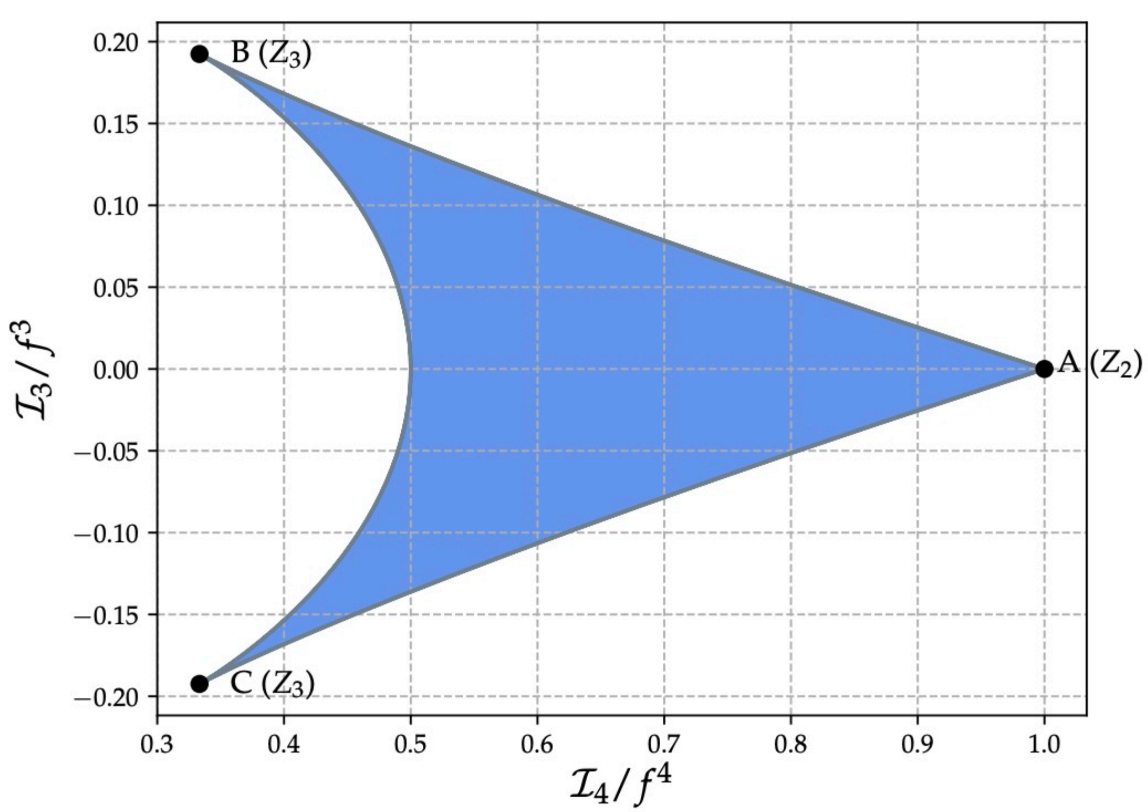
- The maximally natural extrema positions **do not depend** on the coupling constants in the potential
- Maximally natural extrema correspond to vevs that **preserve subgroups** of the discrete symmetry

- For the rest of the talk, I will focus on maximally natural extrema to get a general sense of dGB phenomenology
- There is no guarantee, however, that natural extrema are global minima

The Natural Minima

- The group of transformations that leave positions of the extrema invariant will be the preserved subgroups of the discrete symmetry

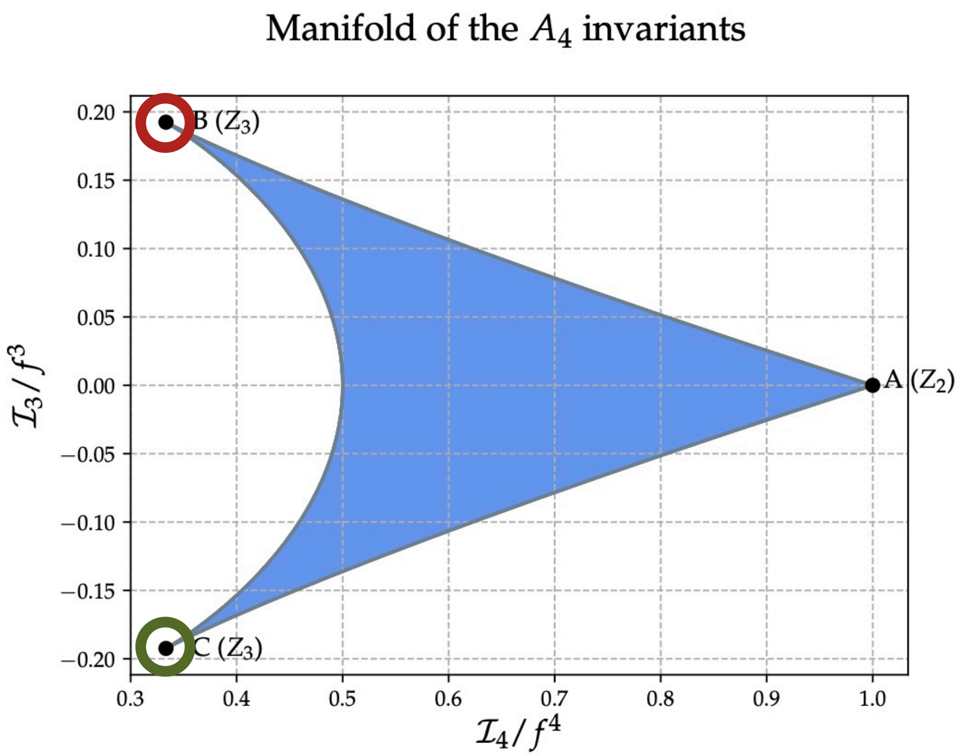
Manifold of the A_4 invariants



Natural singular points for a triplet of A_4

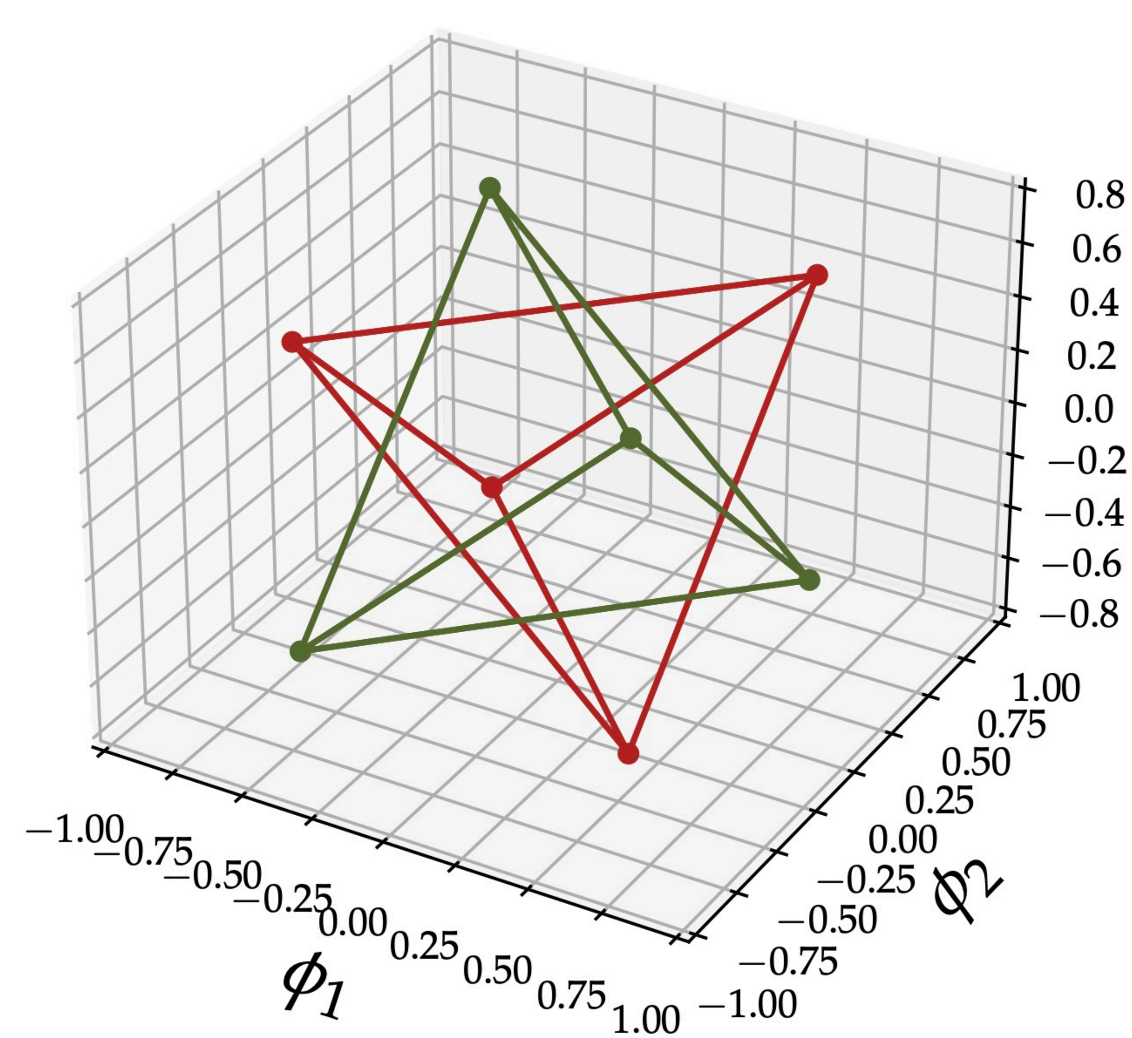
Point	$\mathcal{I}_3$	$\mathcal{I}_4$	ϕ_1	ϕ_2	ϕ_3	Little group	Nature
A	0	1	0 0 ±1	0 ±1 0	±1 0 0	Z_2	Saddles
B	$\frac{1}{3\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\pm\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\mp\frac{1}{\sqrt{3}}$	Z_3	Minima
C	$-\frac{1}{3\sqrt{3}}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\pm\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\mp\frac{1}{\sqrt{3}}$	Z_3	Minima

The Natural Minima



Point	$\mathcal{I}_3$	$\mathcal{I}_4$	ϕ_1	ϕ_2	ϕ_3	Little group	Nature
A	0	1	0 0 ± 1	0 ± 1 0	± 1 0 0	Z_2	Saddles
B	$\frac{1}{3\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\pm\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\mp\frac{1}{\sqrt{3}}$	Z_3	Minima
C	$-\frac{1}{3\sqrt{3}}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\pm\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\mp\frac{1}{\sqrt{3}}$	Z_3	Minima

<u>Position in field space</u>

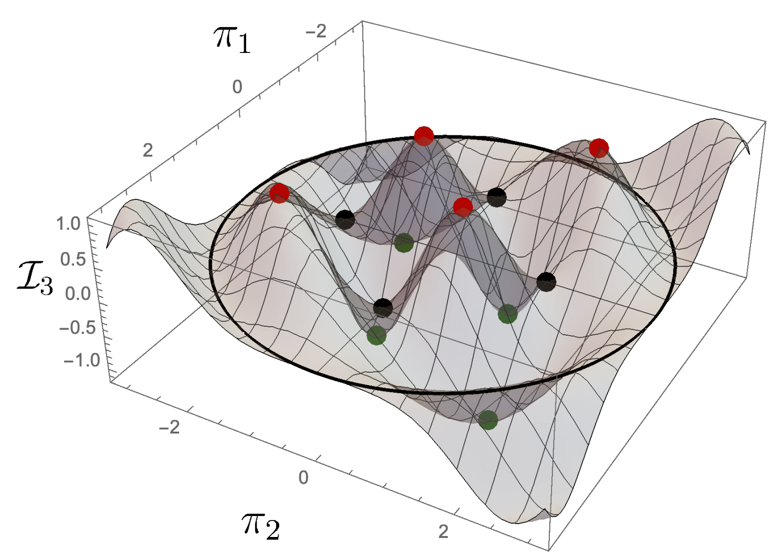


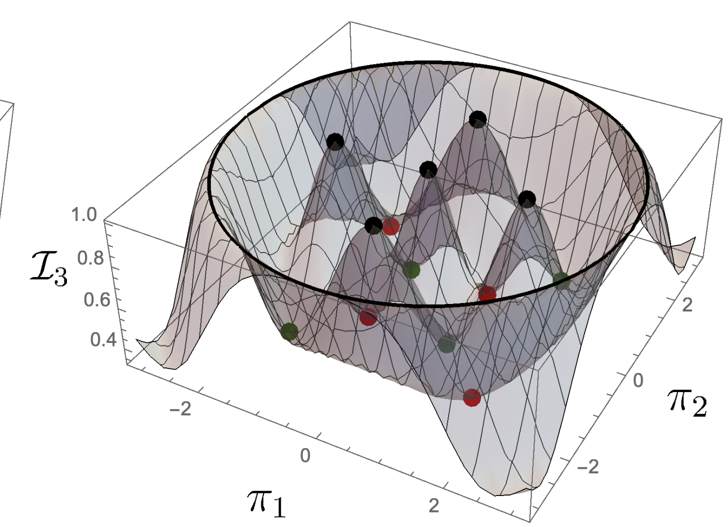
The dGB Fields

- Expanding ϕ around its minimum gives the dGB fields:

$$\phi(\pi_1, \pi_2) = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \exp\left[\frac{1}{f}\begin{pmatrix} 0 & 0 & \pi_1 \\ 0 & 0 & \pi_2 \\ -\pi_1 & -\pi_2 & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

Point	ϕ_1	ϕ_2	ϕ_3	Little group	Nature
A	0	0	± 1	Z_2	Saddles
	0	± 1	0		
	± 1	0	0		
B	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	Z_3	Minima
	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$		
	$\pm\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$\mp\frac{1}{\sqrt{3}}$		
C	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	Z_3	Minima
	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$		
	$\pm\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\mp\frac{1}{\sqrt{3}}$		





The dGB Fields

- The invariants in terms of dGB fields

$$I_3 = \frac{f}{\sqrt{3}}\left[-\frac{f^2}{3} + \pi_1^2 + \pi_2^2 - \frac{1}{3\sqrt{2}f}\left(\pi_1^3 - 3\pi_1\pi_2^2\right) - \frac{17}{24f^2}\left(\pi_1^2 + \pi_2^2\right)^2\right] + \dots$$

$$I_4 = \frac{4f^2}{3}\left[\frac{f^2}{4} + \pi_1^2 + \pi_2^2 + \frac{1}{\sqrt{2}f}\left(\pi_1^3 - 3\pi_1\pi_2^2\right) - \frac{29}{24f^2}\left(\pi_1^2 + \pi_2^2\right)^2\right] + \dots$$

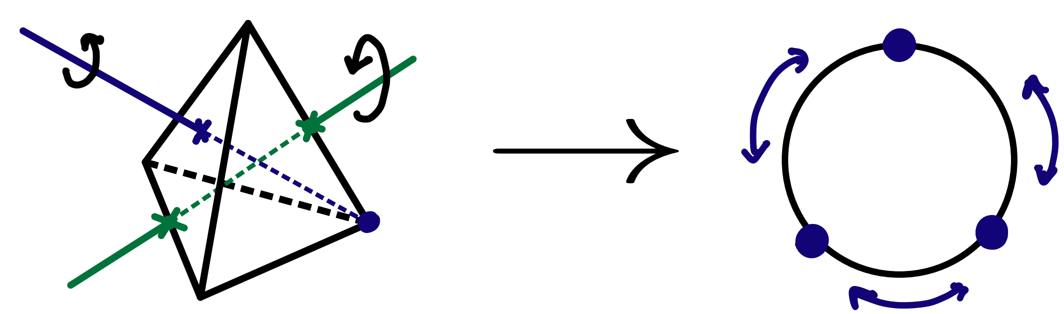
- The Z_3 symmetry is manifest, which can be seen by looking at the invariants of Z_3:

$$\mathcal{I}_2^{(2,Z_3)} = \pi_1^2 + \pi_2^2$$

$$\mathcal{I}_3^{(2,Z_3)} = \pi_1^3 - 3\pi_1\pi_2^2$$

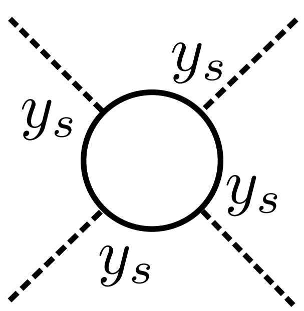
➡ degenerate dGBs

$A_4 \to Z_3$

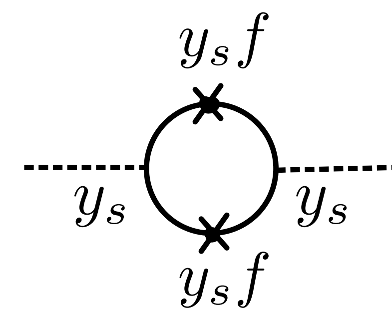


The dGB Mass Protection

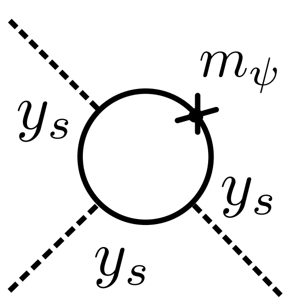
- The leading mass contributions stemming from $SO(3)$-breaking terms are:



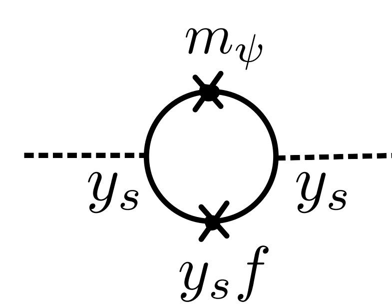
$$\xrightarrow{SO(3)\to SO(2)} \quad \delta m_\pi^2 \ni$$



$$\sim y^4 f^2 \log\left(\frac{m_\psi}{\mu}\right)$$



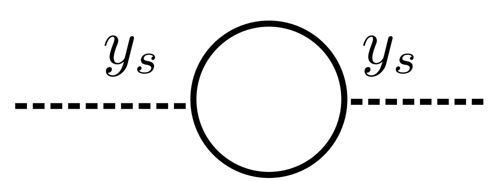
$$\xrightarrow{SO(3)\to SO(2)} \quad \delta m_\pi^2 \ni$$



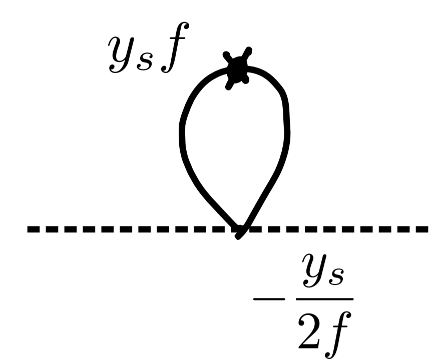
$$\sim y^3 f m_\psi \log\left(\frac{m_\psi}{\mu}\right)$$

- Higher invariants yield either more Yukawa insertions or higher loops

- Guaranteed cancellation:

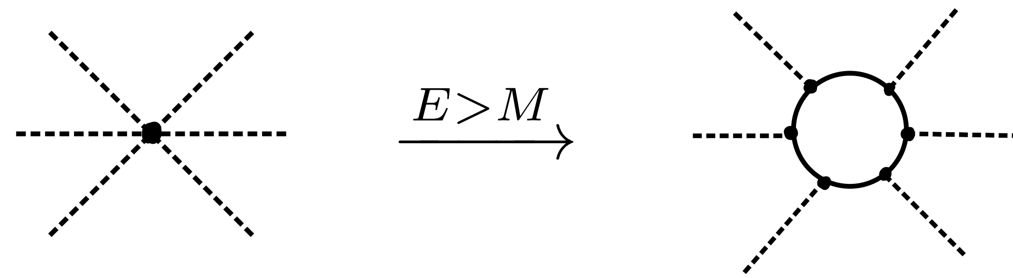


$+$

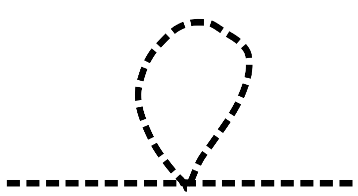


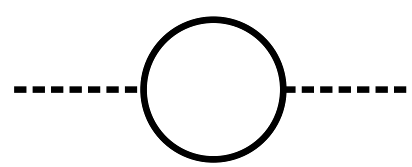
The dGB Mass Protection above the SSB Scale

- Consider again the theory above f with a single fermion with mass M:



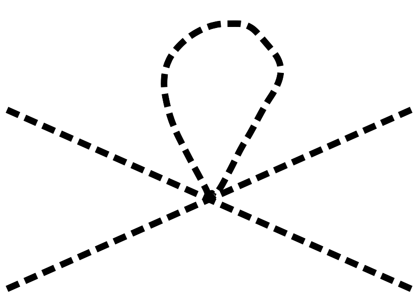
- Above M, the $> \log$ divergent diagrams we can make (at one loop) are:





$\propto \mathcal{I}_2$ ➡ Can't contribute to dGB mass

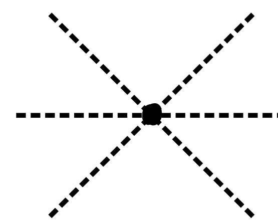
- Below M, we have more operators, and could be worried about diagrams like:



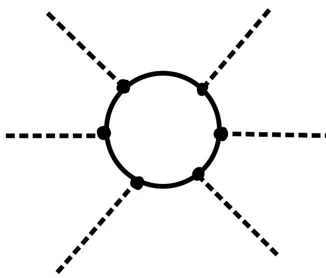
$\sim \hat{c}_6 \, \mathcal{I}_6 \, \frac{M^2}{f^2}$ ➡ Sourcing an $\mathcal{I}_4$, which we know contributes to the dGB mass

The dGB Mass Protection above the SSB Scale

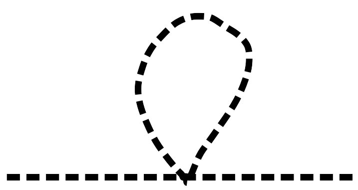
- Consider again the theory above f with a single fermion with mass M:

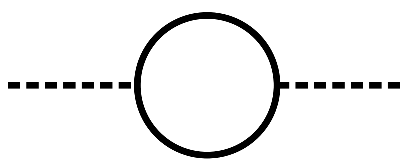


$\xrightarrow{E>M}$



- Above M, the $> \log$ divergent diagrams we can make (at one loop) are:

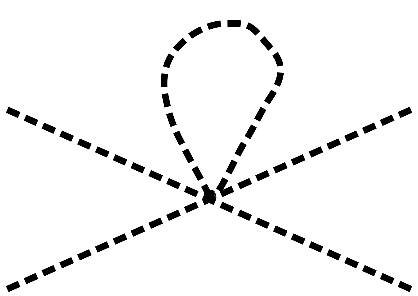




$\propto \mathcal{I}_2$

➡ Can't contribute to dGB mass

- Below M, we have more operators, and could be worried about diagrams like:

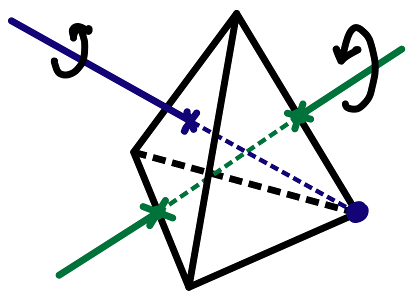


$$\sim \hat{c}_6\, \mathcal{I}_6 \frac{M^2}{f^2} \sim y^6 \left(\frac{f}{M}\right)^2 \frac{M^2}{f^2}$$

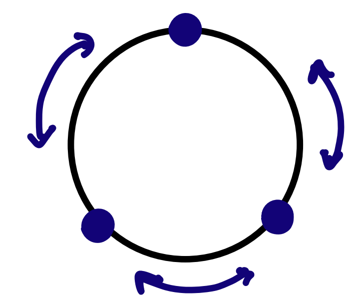
where $\hat{c}_n \sim y^n \left(\frac{f}{M}\right)^{n-4}$

➡ M sensitivity cancels out

Phenomenology of A_4 dGBs



$$A_4 \to Z_3$$



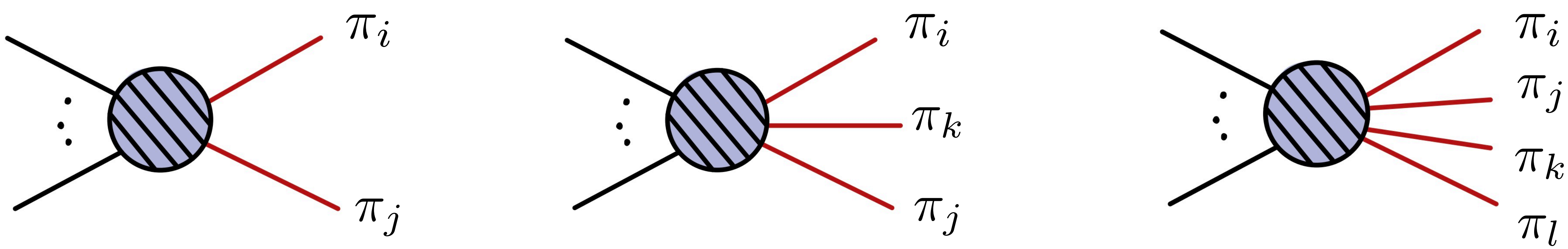
- Two degenerate dGB, guaranteed by the preserved Z_3 symmetry: $$m^2_{\pi_1} = m^2_{\pi_2}$$
- Assume the $\mathcal{I}_3$ operator is the leading term: $$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m} \mathcal{O}^{\text{SM}} \mathcal{I}_3$$
- If the SM production process is uncharged under the discrete group:

$$\mathcal{I}_3 \ni \frac{f}{\sqrt{3}} \left[\underbrace{\pi_1^2 + \pi_2^2}_{\text{2-dGB production}} + \frac{1}{3\sqrt{2} f} \underbrace{\left(\pi_1^3 - 3\pi_1 \pi_2^2\right)}_{\text{3-dGB production}} - \frac{17}{24 f^2} \underbrace{\left(\pi_1^2 + \pi_2^2\right)^2}_{\text{4-dGB production}} \right]$$

Phenomenology of A_4 dGBs

- Because these interactions all stem from the same operator, there is a consistent pattern of production cross sections

$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m}\mathcal{O}^{\text{SM}}\mathcal{I}_3$$



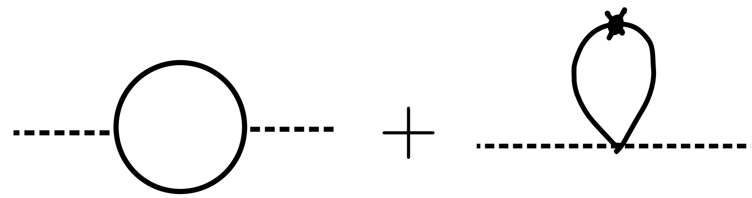
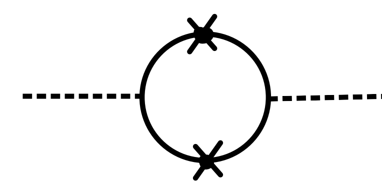
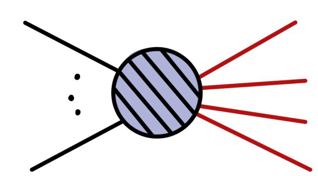
$$\frac{\sigma\left(\text{SM} \to 2\pi\right)}{\sigma\left(\text{SM} \to 3\pi\right)} = 2f^2\frac{\Pi_2}{\Pi_3} \quad = 64\pi^2\frac{f^2}{E_{\text{CM}}^2}$$

$$\frac{\sigma\left(\text{SM} \to 3\pi\right)}{\sigma\left(\text{SM} \to 4\pi\right)} = \frac{36f^2}{19(17)^2}\frac{\Pi_3}{\Pi_4} \quad = \frac{6(24\pi)^2}{19(17)^2}\frac{f^2}{E_{\text{CM}}^2}$$

- This cross section information tells us about the A_4 symmetry, not just the preserved Z_3

Low (2014)

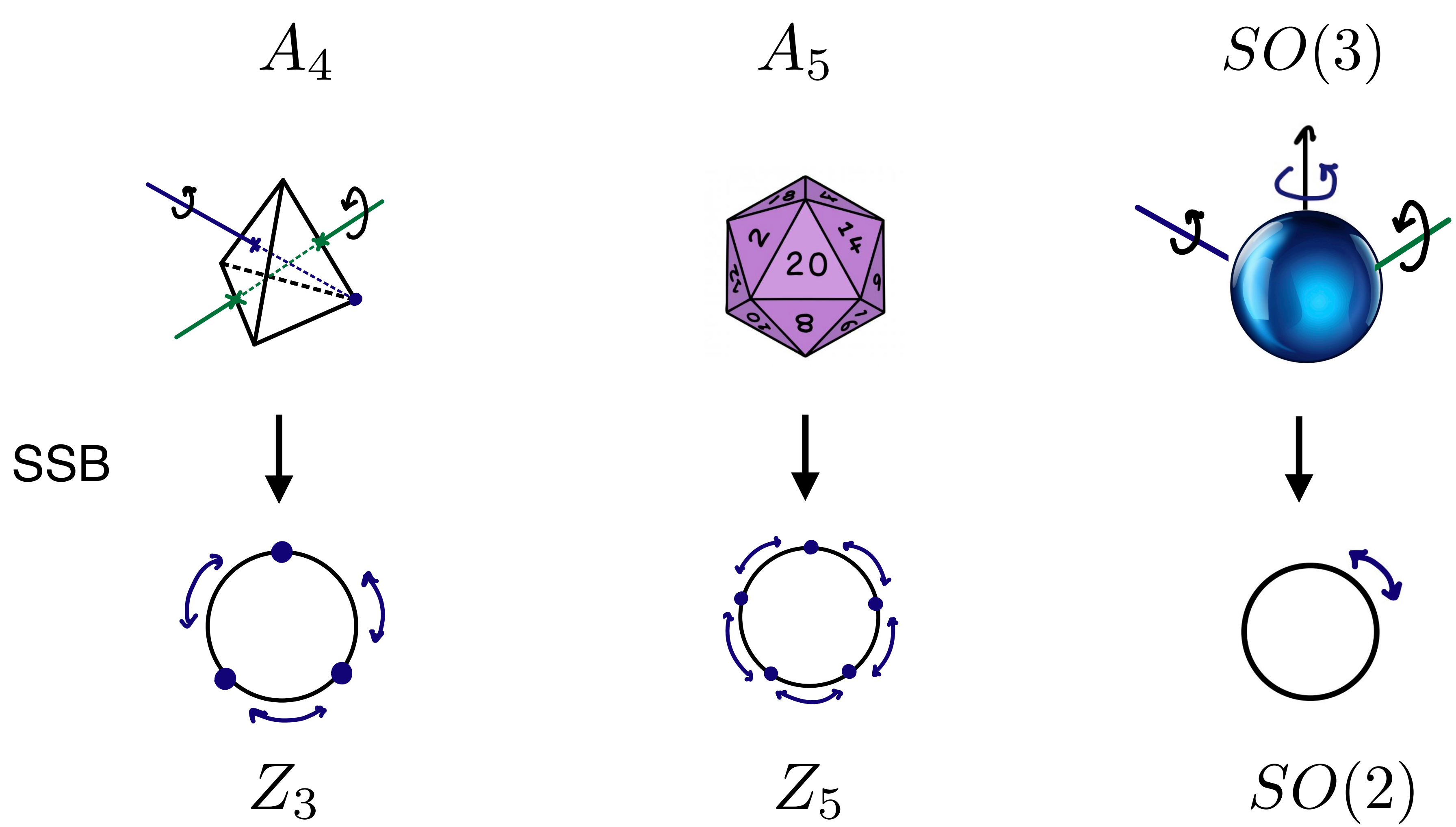
$A_4 \to Z_3$ dGB Summary

- Naturally light, nonzero masses:
  - LO quadratically divergent contributions cancel:  $= 0$
  - Nonzero mass generated even with **exact symmetry**  $\sim y^4 f^2 \log\left(\frac{m_\psi}{\mu}\right)$
- Smoking gun signal: simultaneous invisible production 
- Access to the full UV symmetry group by measuring ratios of invisible production

$$\frac{\sigma\left(\mathrm{SM} \to 3\pi\right)}{\sigma\left(\mathrm{SM} \to 4\pi\right)} = \frac{36 f^2}{19(17)^2} \frac{\Pi_3}{\Pi_4}$$

This ratio is a result of the A_4 symmetry active **above the SSB scale**

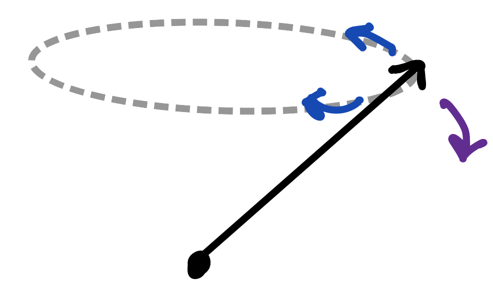
Higher Non-Abelian Discrete Symmetry Example: A_5



The Triplet of A_5

Even permutations of:
$(x_1, x_2, x_3, x_4, x_5)$

- Irreducible representations of A_5: $1 \quad 3 \quad 3' \quad 4 \quad 5$
- A_4 is a subgroup of A_5
- Consider the dGB's stemming from a scalar field in the **triplet** representation of A_5



The Triplet of A_5

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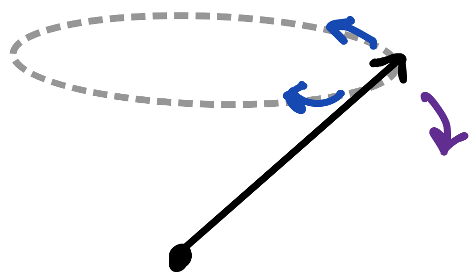
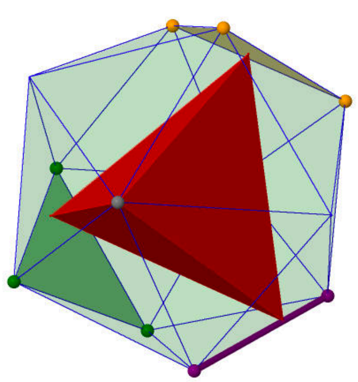


- The primary invariants of the triplet of A_5:

$$\mathcal{I}_2^{(\mathbf{3},A_5)} = \mathcal{I}_2$$

$$\mathcal{I}_6^{(\mathbf{3},A_5)} = 22\,\mathcal{I}_3^2 + \mathcal{I}_2\mathcal{I}_4 - 2\sqrt{5}\mathcal{I}_6$$

$$\mathcal{I}_{10}^{(\mathbf{3},A_5)} = \mathcal{I}_2\mathcal{I}_4^2 + 38\,\mathcal{I}_3^2\mathcal{I}_4 - \frac{7}{11}\mathcal{I}_2^3\mathcal{I}_4 - \frac{128}{11\sqrt{5}}\mathcal{I}_2^2\mathcal{I}_6 + \frac{6}{\sqrt{5}}\mathcal{I}_4\mathcal{I}_6$$



where $(\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4, \mathcal{I}_6)$ are the invariants of A_4

The Triplet of A_5 Potential

- The most general potential of the dGB's:

$$V_{\rm dGB} = f^2\Lambda^2\left[\hat{c}_6\frac{\mathcal{I}_6}{f^6} + \hat{c}_{10}\frac{\mathcal{I}_{10}}{f^{10}} + \hat{c}_{12}\frac{\mathcal{I}_6^2}{f^{12}} + \hat{c}_{15}\frac{\mathcal{I}_{15}}{f^{15}} + ...\right]$$

- Note that the A_5 symmetry forbids $\hat{c}_n$ for $n < 6$

➡ All terms in the potential come from higher dimensional operators

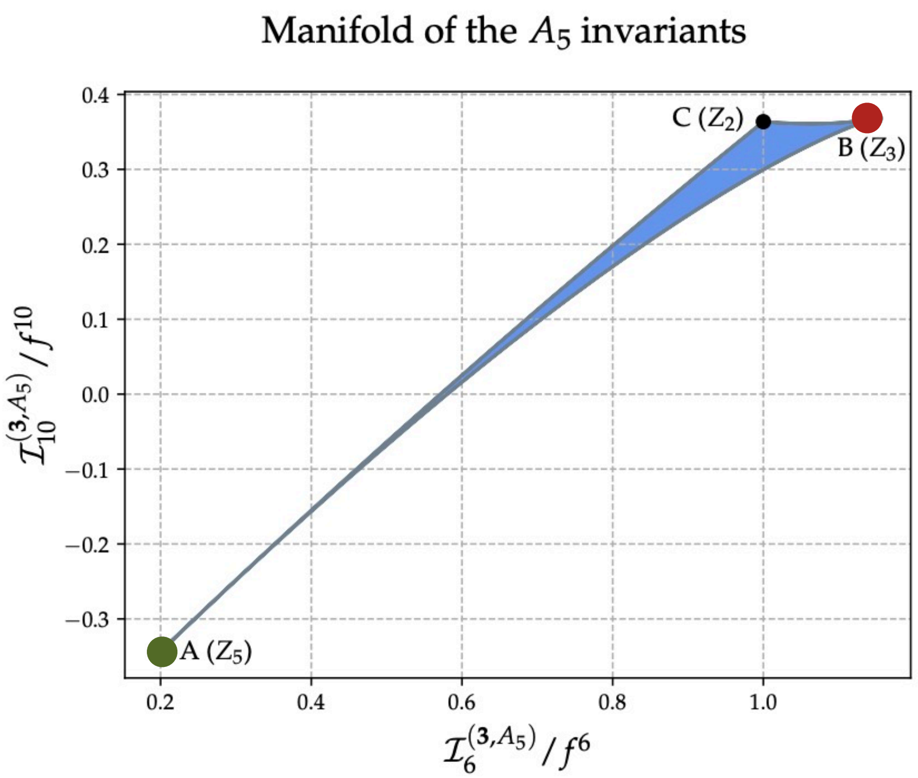
➡ This leads to an enhanced suppression for m_π :

$$m_\pi^2 \sim \hat{c}_6 f^2 \qquad\qquad \hat{c}_n \sim y^n \left(\frac{f}{M}\right)^{n-4}$$

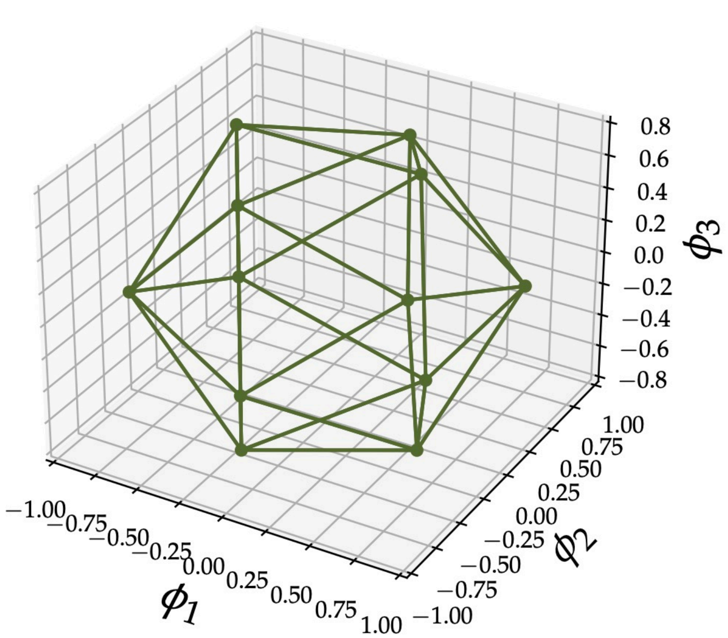
More suppressed   Still tied to f

Natural Minima of the Triplet of A_5

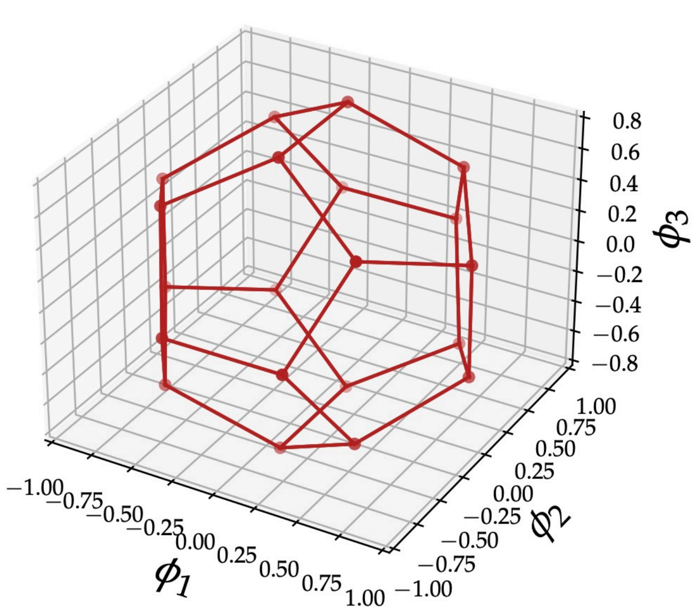
- The manifold spanned by $\mathcal{I}_6$ and $\mathcal{I}_{10}$ is again bounded, yielding a set of maximally natural minima



Z_5 invariant points (A)



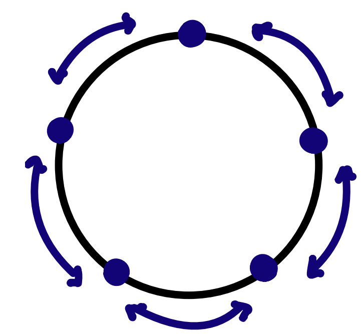
Z_3 invariant points (B)



(Point C is a saddlepoint)

Phenomenology of $A_5 \to Z_5$

$$A_5 \to Z_5$$



- Two degenerate dGB, guaranteed by the preserved Z_5 symmetry: $$m_{\pi_1}^2 = m_{\pi_2}^2$$

- Assume the $\mathcal{I}_6$ operator is the leading term: $$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m}\mathcal{O}^{\text{SM}}\mathcal{I}_6^{(\mathbf{3},A_5)} + \ldots$$

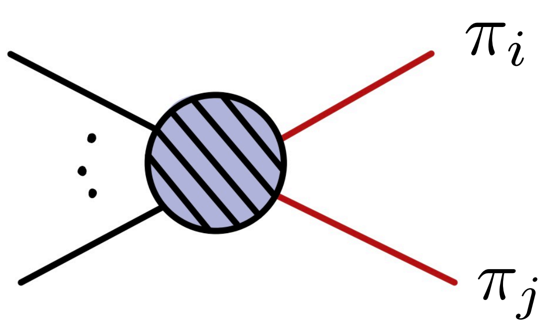
- If the SM production process is uncharged under the discrete group:

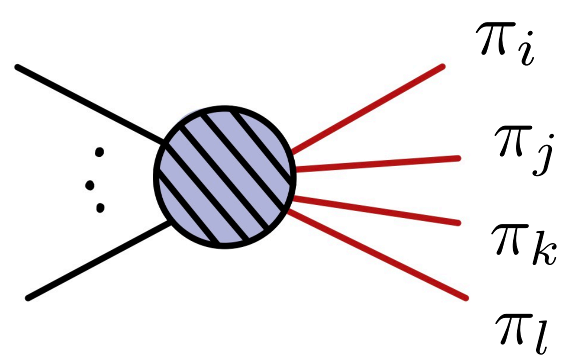
$$\mathcal{I}_6^{(\mathbf{3},A_5)} = \frac{32}{5}f^4\left[\frac{f^2}{32} + \underbrace{\pi_1^2 + \pi_2^2}_{\text{2-dGB production}} - \frac{31}{12f^2}\underbrace{\left(\pi_1^2+\pi_2^2\right)^2}_{\text{4-dGB production}} - \frac{1}{4f^3}\underbrace{\left(\pi_1^5 - 10\pi_1^3\pi_2^2 + 5\pi_1\pi_2^4\right)}_{\text{5-dGB production}}\right]$$

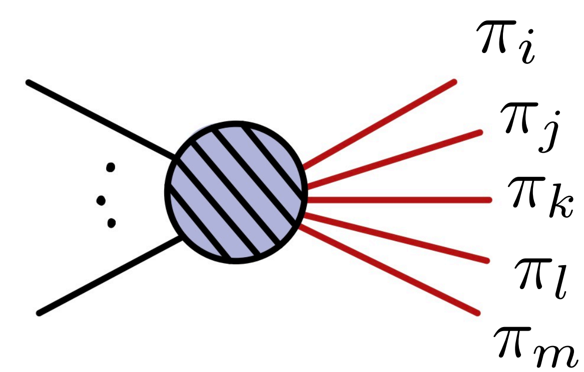
Phenomenology of $A_5 \to Z_5$

- Because these interactions all stem from the same operator, there is a consistent pattern of production cross sections

$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m} \mathcal{O}^{\text{SM}} \mathcal{I}_6^{(\mathbf{3}, A_5)}$$







$$\frac{\sigma(\text{SM} \to 4\pi)}{\sigma(\text{SM} \to 5\pi)} = \frac{19(31)^2 f^2}{3(45)^2} \frac{\Pi_4}{\Pi_5} = \frac{19(31)^2 (8\pi)^2}{45^2} \frac{f^2}{E_{\text{CM}}^2}$$

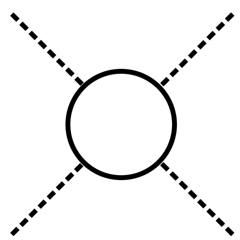
$$\frac{\sigma(\text{SM} \to 2\pi)}{\sigma(\text{SM} \to 4\pi)} = \frac{18 f^4}{19(31)^2} \frac{\Pi_2}{\Pi_4} = \frac{216(4\pi)^4}{19(31)^2} \frac{f^4}{E_{\text{CM}}^4}$$

- This cross section information tells us about the A_5 symmetry, not just the preserved Z_5

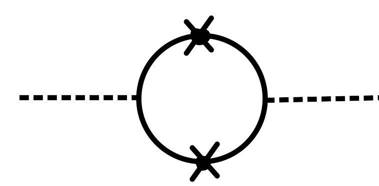
A_4 vs. A_5

- Both dGB's are protected at LO from quadratic divergences, but A_5 shows an even more enhanced mass suppression

A_4 : $\mathcal{I}_4$

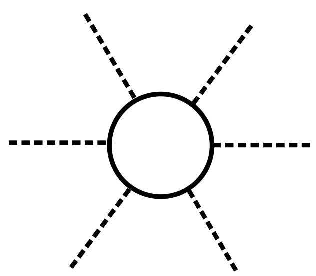


$\xrightarrow{SO(3)\to SO(2)}$

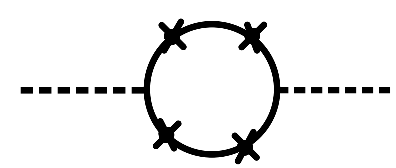


$\sim y^4 f^2 \log\left(\frac{m_\psi}{\mu}\right)$

A_5 : $\mathcal{I}_6$



$\xrightarrow{SO(3)\to SO(2)}$

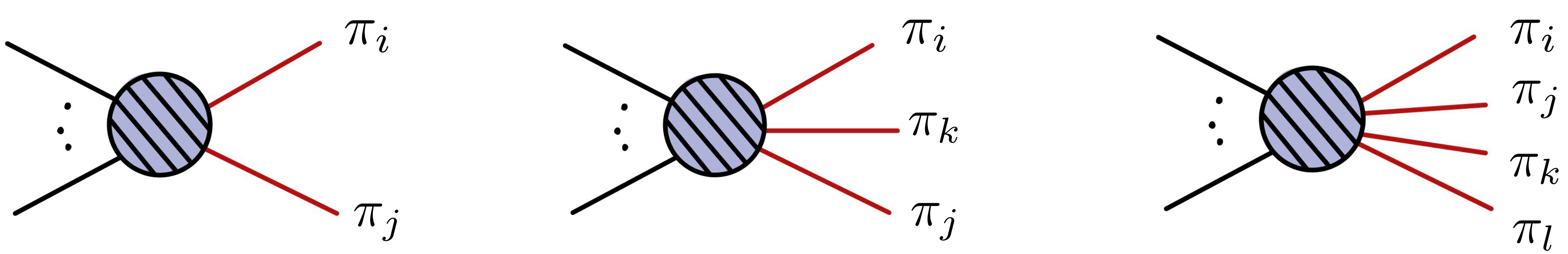


$\sim y^6 \left(\frac{f}{M}\right)^2 f^2$

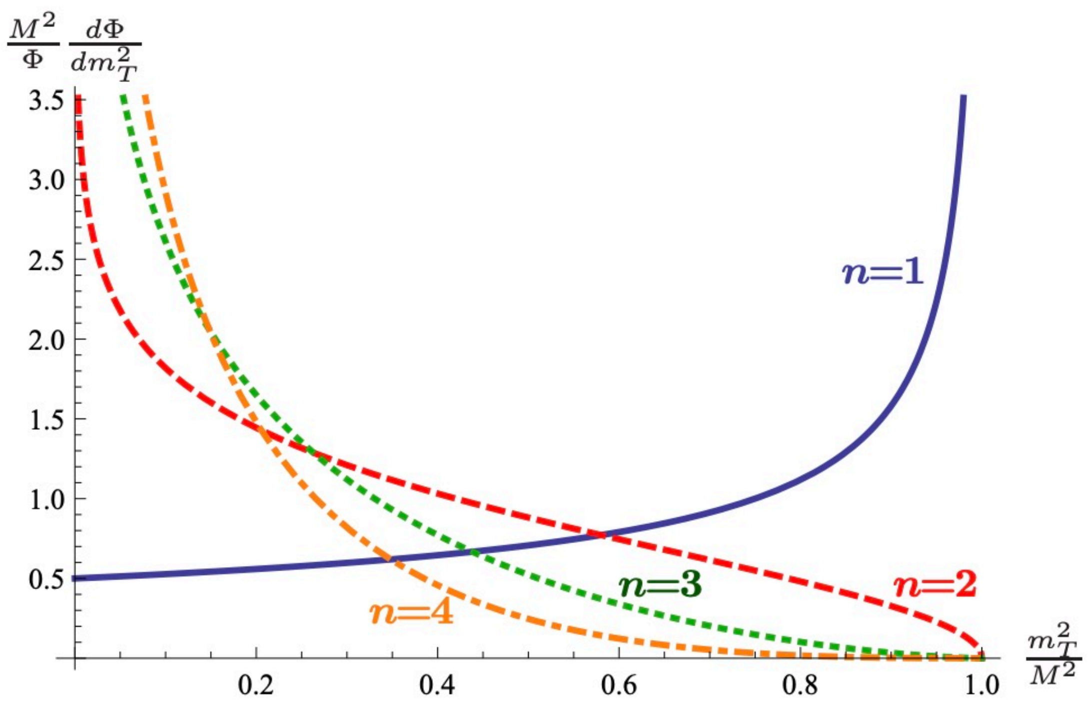
- Both dGB's predict a pattern of invisible particle production:

$$A_4 : \quad \frac{\sigma(\mathrm{SM} \to 2\pi)}{\sigma(\mathrm{SM} \to 4\pi)} = \frac{72 f^4}{19(17)^2} \frac{\Pi_2}{\Pi_4} \qquad A_5 : \quad \frac{\sigma(\mathrm{SM} \to 2\pi)}{\sigma(\mathrm{SM} \to 4\pi)} = \frac{18 f^4}{19(31)^2} \frac{\Pi_2}{\Pi_4}$$

Counting Invisibles



Counting Invisible Particles in a Collider

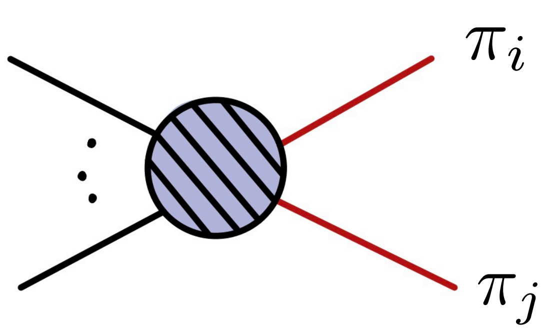


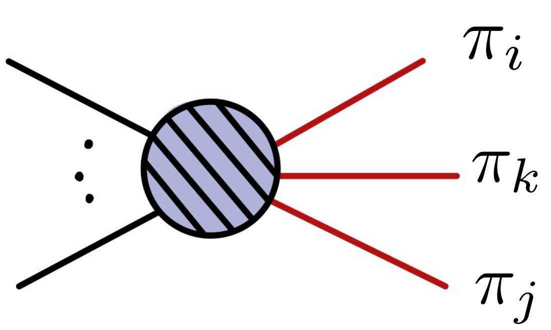
Plot lifted from Giudice, Gripaios, Mahbubani, arXiv:1108.1800

- In a collider, we can disentangle multi-particle production via tails of kinematic variables similar to

$$m_T^2 = m_V^2 + 2\left(\sqrt{\not{p}_T^2(p_T^2 + m_V^2)} - \not{p}_T \cdot p_T\right)$$

- Naively, this means we can detect these individual processes and measure their production rates



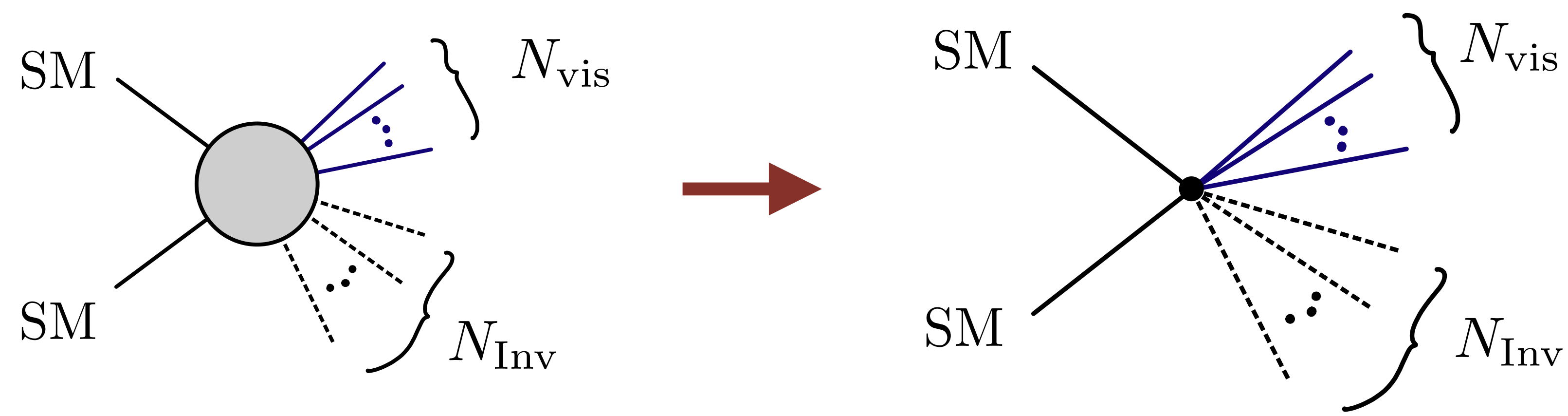


$$\frac{\sigma\left(\mathrm{SM} \to 2\pi\right)}{\sigma\left(\mathrm{SM} \to 3\pi\right)} = 64\pi^2 \frac{f^2}{E_{\mathrm{CM}}^2}$$

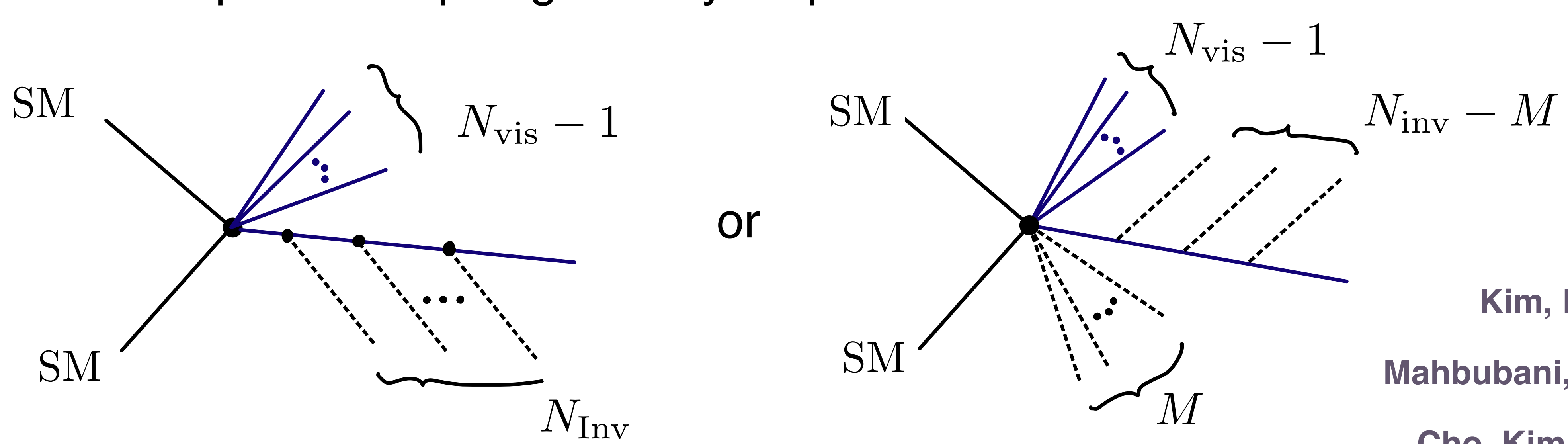
Counting Invisible Particles in a Collider

- These distributions were based on the assumption of single vertex production of both visible and invisibles

Giudice, Gripaios, Mahbubani, (2011)



- More complicated topologies may be present



Kim, Matchev, Park (2015)

Mahbubani, Matchev, Park (2012)

Cho, Kim, Matchev, Park (2012)

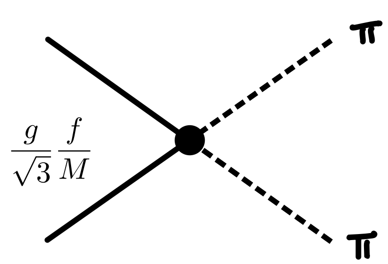
Counting Invisibles

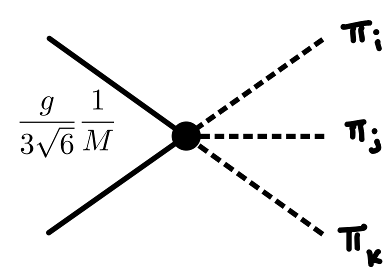
- The smoking gun signal for the A_4 symmetry

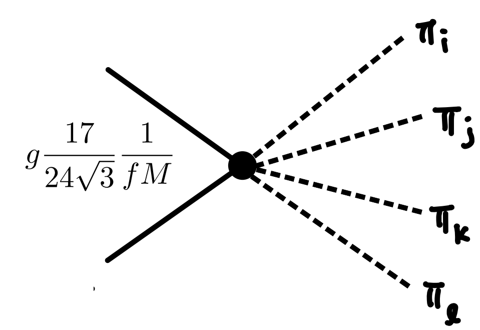
$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m}\mathcal{O}^{\text{SM}}\mathcal{I}_3$$

(Assuming $M \gg f$, this operator dominates)

$$I_3 = \frac{f}{\sqrt{3}}\left[\underbrace{\pi_1^2 + \pi_2^2}_{\text{2-dGB production}} - \frac{1}{3\sqrt{2}f}\underbrace{\left(\pi_1^3 - 3\pi_1\pi_2^2\right)}_{\text{3-dGB production}} - \frac{17}{24f^2}\underbrace{\left(\pi_1^2 + \pi_2^2\right)^2}_{\text{4-dGB production}}\right] + \ldots$$



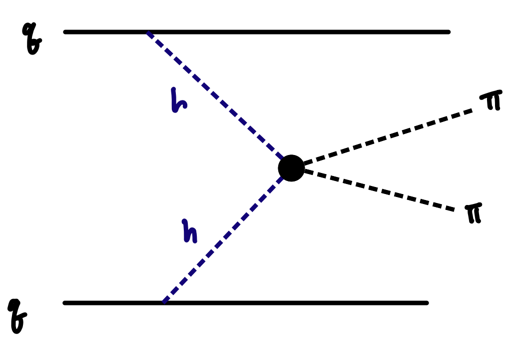


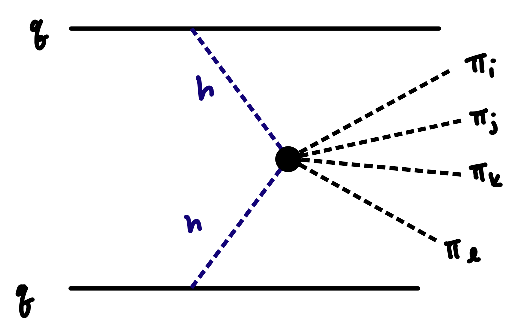


- The symmetry informs the couplings of these vertices, not necessarily how many dGB's are in the final state for the full process

Counting Invisibles: an Example

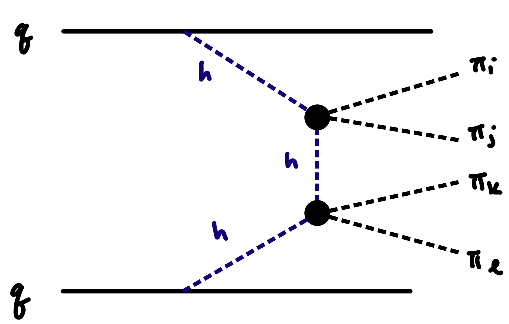
- One possible production mechanism:

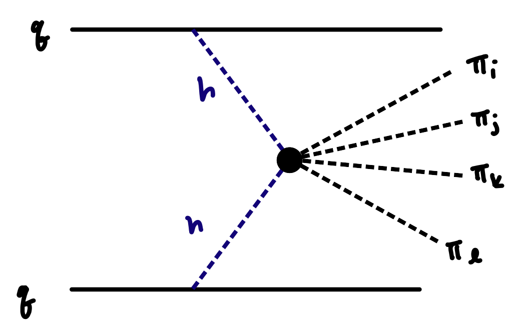




$$\rightarrow \frac{\sigma(\mathrm{SM} \to 2\pi)}{\sigma(\mathrm{SM} \to 4\pi)} \rightarrow A_4$$

- Can this reliably be extracted when we also have the following?





Both have four dGB's in the final state!

- To access the UV discrete symmetry, we want to distinguish **vertices**

Ambiguity for Invisible Production

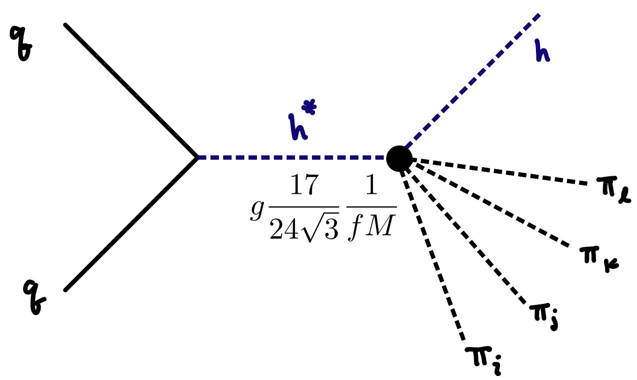
- Compare the following:

These rough estimates for the amplitudes are comparable when, for example:

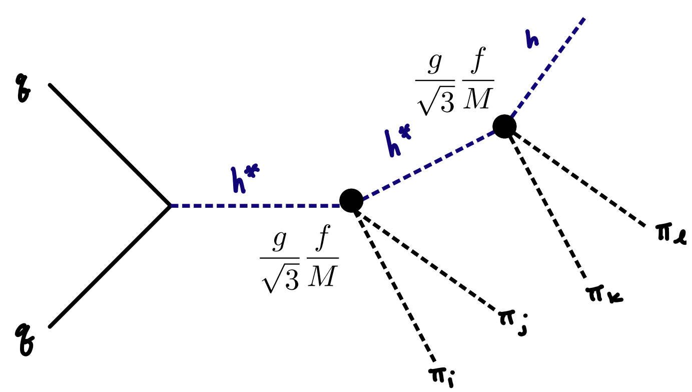
$$f \sim \text{TeV}$$

$$M \sim 10 \text{ TeV}$$

$$g \sim 0.1$$



$$\sim g\frac{1}{fMm_h^2}$$



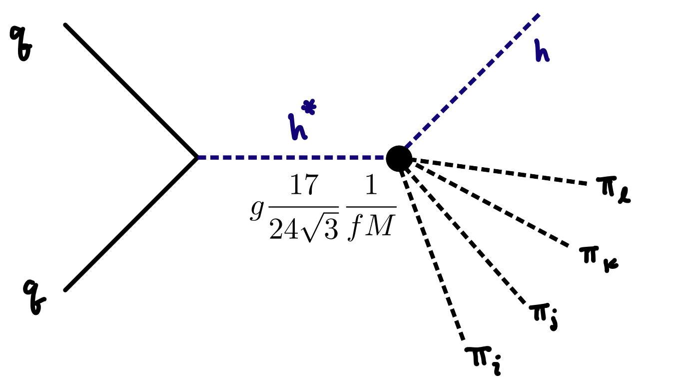
$$\sim g^2\frac{f^2}{M^2m_h^4}$$

- For just one visible particle in the final state, these two processes are distinguishable just from studying the MET distributions

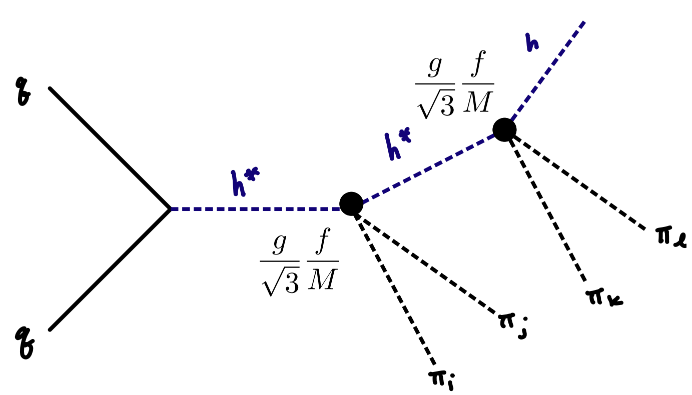
Ambiguity for Invisible Production

- Compare the following:

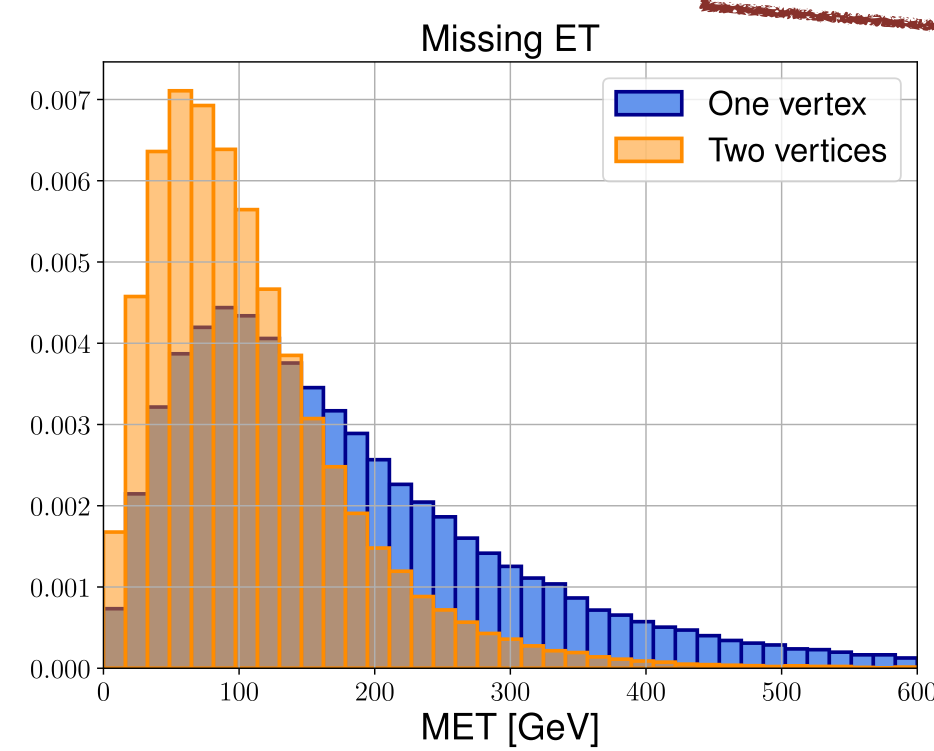
Preliminary



$$\sim g\frac{1}{fMm_h^2}$$



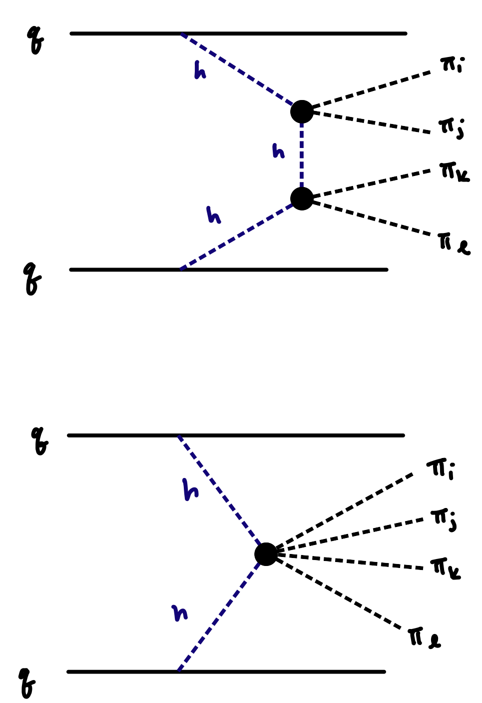
$$\sim g^2\frac{f^2}{M^2m_h^4}$$

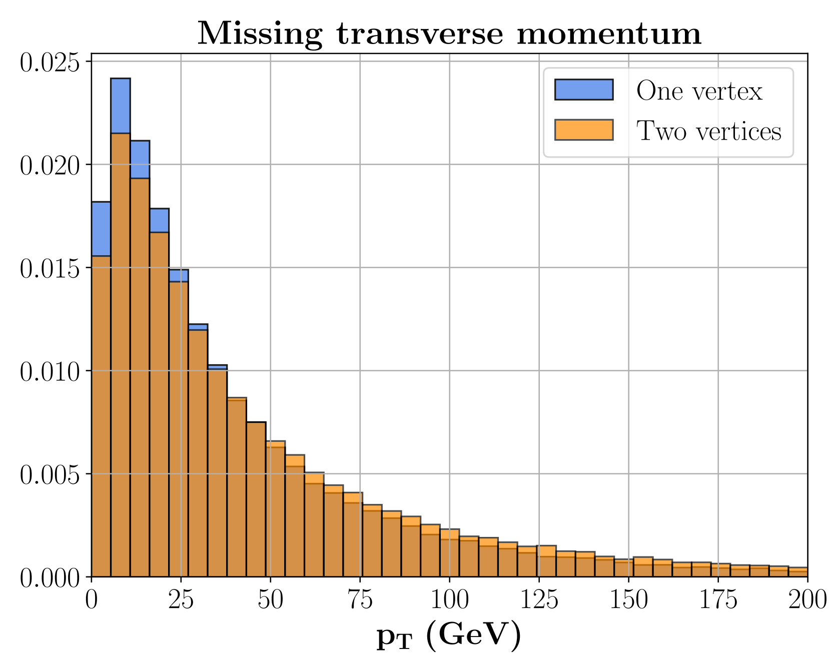


- For just one visible particle in the final state, these two processes are distinguishable just from studying the MET distributions

Ambiguity for Invisible Production

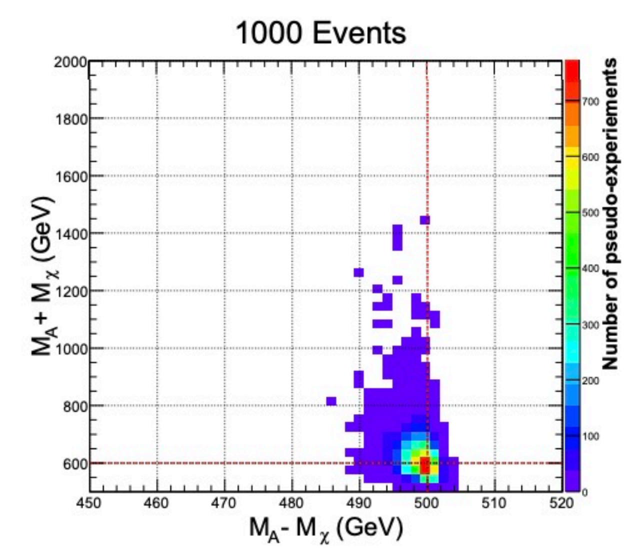
- When there are >1 visible particles in the final state, this becomes more difficult





(Preliminary)

➡ Indistinguishable



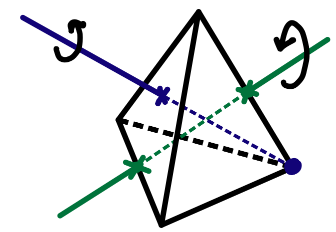
Plot lifted from Cho, Kim, Matchev, Park, arXiv:1206.1546

- Currently looking at p_T distributions for individual particles in order to obtain a 2D scatter distribution

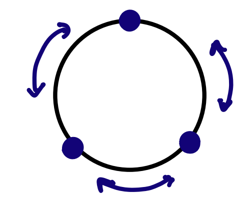
➡ An ideal classification problem for a NN

Conclusion

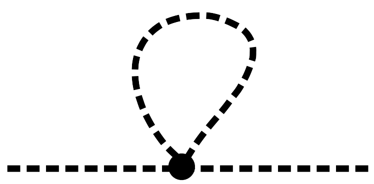
- Nonlinearly realized discrete symmetries produce **discrete Goldstone bosons**



$A_4 \rightarrow Z_3$

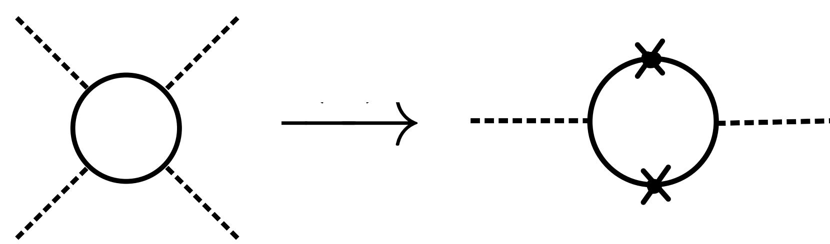


- dGBs have enhanced protection from quadratic divergences



$\mathcal{I}_4$

$\Rightarrow \mathcal{I}_2 = f^2$



$\sim y^4 f^2 \log\left(\frac{m_\psi}{\mu}\right)$

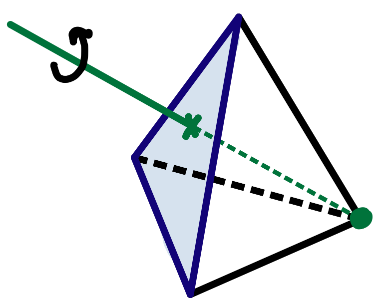
- Wrote down an EFT of dGBs and classified its general features
    - Degenerate dGBs reflecting preserved low energy symmetries
    - Multi-particle invisible production
    - Cross section ratios give access to nonlinearly realized symmetry *(if we can count invisibles…)*

Thank you!

Back-up Slides

More Discrete Groups: A_N , S_N

A_4 Tetrahedron



Even permutations of:
(x_1, x_2, x_3, x_4)

A_5 Icosahedron

Even permutations of:
$(x_1, x_2, x_3, x_4, x_5)$

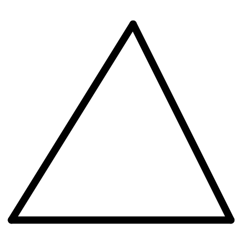
Higher A_N

…

Even permutations of N objects:

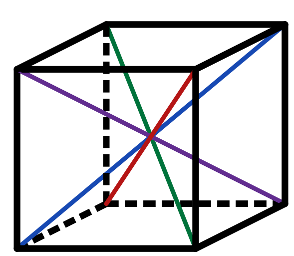
$(x_1, x_2, ..., x_N)$

S_3 Triangle

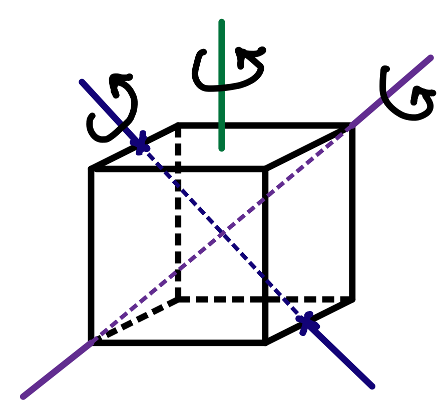


All permutations of:
(x_1, x_2, x_3)

S_4 Cube



All permutations of:
(x_1, x_2, x_3, x_4)



Higher S_N

…

All permutations of N objects:

$(x_1, x_2, ..., x_N)$

Higher Non-Abelian Discrete Symmetry Examples

The Quadruplet of A_5

Even permutations of:
$(x_1, x_2, x_3, x_4, x_5)$

- Irreducible representations of A_5: $\mathbf{1} \quad \mathbf{3} \quad \mathbf{3'} \quad \mathbf{4} \quad \mathbf{5}$

$$\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)$$

- This case will have a non-Abelian preserved symmetry

- The primary invariants of the quadruplet of A_5:

$$\mathcal{I}_2^{(\mathbf{4},A_5)} = \mathcal{I}_2 + \phi_4^2$$

$$\mathcal{I}_3^{(\mathbf{4},A_5)} = \mathcal{I}_3 - \frac{\phi_4}{2\sqrt{5}}\mathcal{I}_2 + \frac{\phi_4^3}{2\sqrt{5}}$$

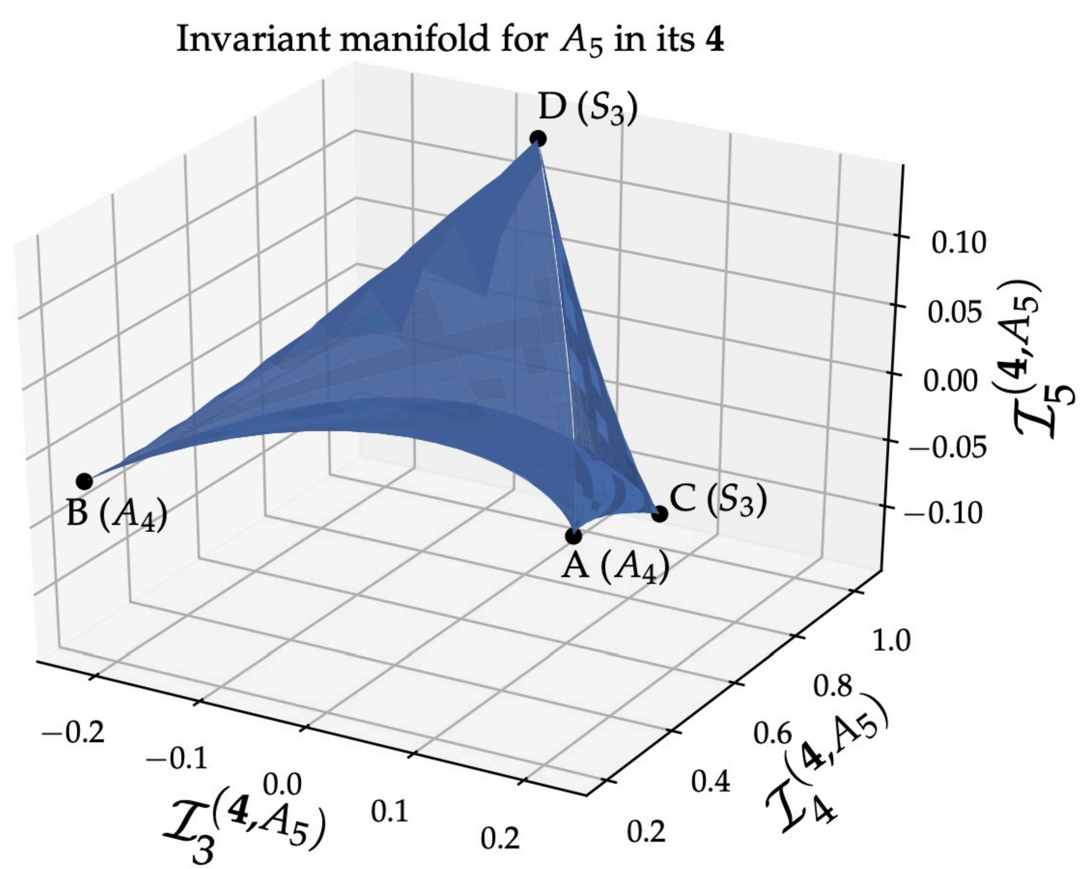
$$\mathcal{I}_4^{(\mathbf{4},A_5)} = \mathcal{I}_4 + \frac{12}{\sqrt{5}}\mathcal{I}_3\phi_4 + \frac{12}{5}\mathcal{I}_2\phi_4^2 + \frac{\phi_4^4}{5}$$

$$\mathcal{I}_5^{(\mathbf{4},A_5)} = \mathcal{I}_4\phi_4 - \frac{1}{2}\mathcal{I}_2\phi_4 - \frac{4}{\sqrt{5}}\mathcal{I}_3\phi_4^2 - \frac{\phi_4^3}{5}\mathcal{I}_2 + \frac{\mathcal{I}_4^5}{50}$$

where $(\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4)$ are the primary invariants of A_4

Natural Minima for the Quadruplet of A_5

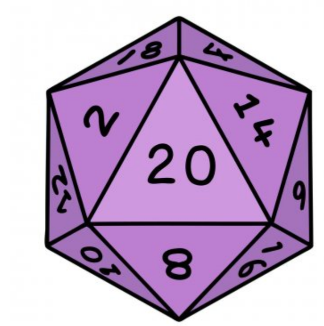
- The manifold is spanned by $\mathcal{I}_{3,4,5}$
- The maximally natural extrema now have non-Abelian symmetries remaining in the low energy spectrum



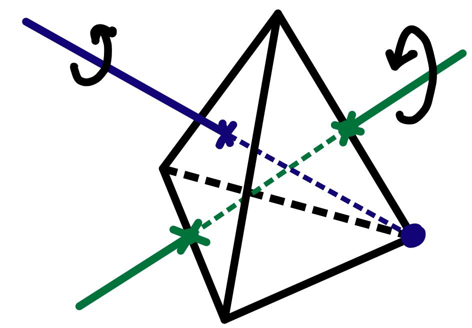
Natural singular points for a quadruplet of A_5

Point	$\mathcal{I}_3$	$\mathcal{I}_4$	$\mathcal{I}_5$	ϕ_1	ϕ_2 (*Representatives*)	ϕ_3	ϕ_4	Little group	Nature
A	$\frac{1}{2\sqrt{5}}$	$\frac{1}{5}$	$\frac{1}{50}$	0 $-\frac{\sqrt{5}}{4}$	0 $-\frac{\sqrt{5}}{4}$	0 $-\frac{\sqrt{5}}{4}$	1 $\frac{1}{4}$	A_4	Minima
B	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{5}$	$-\frac{1}{50}$	0 $\frac{\sqrt{5}}{4}$	0 $\frac{\sqrt{5}}{4}$	0 $\frac{\sqrt{5}}{4}$	-1 $-\frac{1}{4}$		
C	$\frac{1}{3\sqrt{30}}$	$\frac{31}{30}$	$-\frac{4}{25}\sqrt{\frac{2}{3}}$	$\sqrt{\frac{5}{24}}$ $-\sqrt{\frac{5}{6}}$	$\sqrt{\frac{5}{24}}$ 0	$\sqrt{\frac{5}{24}}$ 0	$\sqrt{\frac{3}{8}}$ $-\frac{1}{\sqrt{6}}$	S_3	Saddles
D	$-\frac{1}{3\sqrt{30}}$	$\frac{31}{30}$	$\frac{4}{25}\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{5}{24}}$ $\sqrt{\frac{5}{6}}$	$-\sqrt{\frac{5}{24}}$ 0	$-\sqrt{\frac{5}{24}}$ 0	$-\sqrt{\frac{3}{8}}$ $\frac{1}{\sqrt{6}}$		

Phenomenology of $A_5 \to A_4$



$$A_5 \to A_4$$



- Three degenerate dGBs, guaranteed by the preserved A_4 symmetry:

$$m_{\pi_1} = m_{\pi_2} = m_{\pi_3}$$

- Assume the $\mathcal{I}_3$ operator is the leading term:

$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m} \mathcal{O}^{\text{SM}} \mathcal{I}_3$$

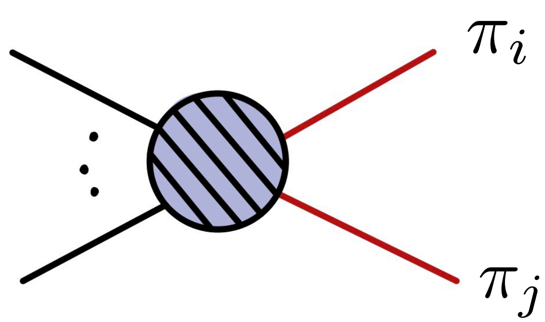
- If the SM production process is uncharged under the discrete group:

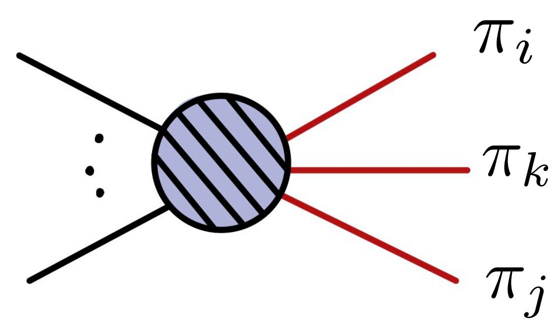
$$\mathcal{I}_3^{(\mathbf{4}, A_5)} = \frac{\sqrt{5} f}{4} \left[-\frac{2 f^2}{5} + \underbrace{\left(\pi_1^2 + \pi_2^2 + \pi_3^2\right)}_{\text{2-dGB production}} - \underbrace{\frac{4}{\sqrt{5} f} \pi_1 \pi_2 \pi_3}_{\text{3-dGB production}} - \underbrace{\frac{41}{60 f^2} \left(\pi_1^2 + \pi_2^2 + \pi_3^2\right)^2}_{\text{4-dGB production}} \right]$$

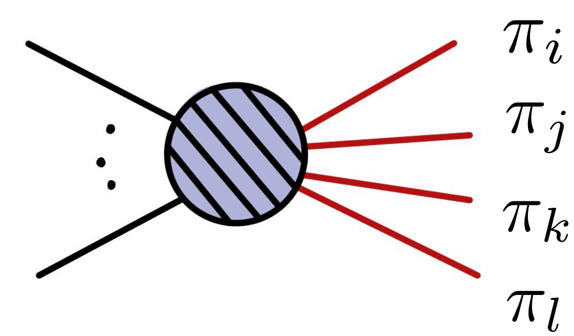
Phenomenology of $A_5 \to A_4$

- Because these interactions all stem from the same operator, there is a consistent pattern of production cross sections

$$\mathcal{L}_{\text{int}} \propto \frac{1}{M^m}\mathcal{O}^{\text{SM}}\mathcal{I}_3^{(\mathbf{4},A_5)}$$







$$\frac{\sigma(\text{SM} \to 3\pi)}{\sigma(\text{SM} \to 4\pi)} = \frac{6f^2}{(41)^2}\frac{\Pi_3}{\Pi_4} = \left(\frac{24\pi}{41}\right)^2 \frac{f^2}{E_{\text{CM}}^2}$$

$$\frac{\sigma(\text{SM} \to 2\pi)}{\sigma(\text{SM} \to 3\pi)} = \frac{15f^2}{4}\frac{\Pi_2}{\Pi_3} = 120\pi^2 \frac{f^2}{E_{\text{CM}}^2}$$

- This cross section information tells us about the A_5 symmetry, not just the preserved A_4